15. Identification of closed loop systems

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Introduction

- Many systems have to be run under feedback control
  - The plant is unstable
  - The plant has to be controlled for production, economic, or safety reasons
  - The plant contains inherent feedback mechanisms ($e.g.$, biological systems)

- How the open loop system can be identified when it must operate under feedback control?

Identifiability considerations

We will use the following model in the analysis

$$
\begin{aligned}
y(t) &= G(q^{-1})u(t) + H(q^{-1})e(t), \quad \mathbf{E}\, e(t)e(t)^* = \Lambda\delta_{t,s} \\
u(t) &= -F(q^{-1})y(t) + L(q^{-1})\nu(t)
\end{aligned}
$$

- $\nu(t)$ can be a reference, a setpoint, or noise entering the regulator
- The input $u(t)$ depends on $y(t)$ (hence, dependence between $u(t)$ and $e(t)$)
- The aim with control is to minimize the deviation between $y(t)$ and the reference signal $\nu(t)$. Good control implies a small value of $u(t)$
- System identification requires good excitation, implying large variations in $u(t)$. Hence, there is a conflict with the previous aspect
- The frequency contents of the input is limited by the true system

Assumptions

- The open loop system is strictly proper ($G(0) = 0$). Otherwise, an algebraic loop occurs
- The subsystems from ν and e to y of the closed loop system are asymptotically stable and has no unstable hidden mode:

$$L(q^{-1}), H(q^{-1}), \text{ and } [I + G(q^{-1})F(q^{-1})]^{-1} \text{ are asymptotically stable}$$

- $\nu(t)$ is stationary and persistently exciting of a sufficient order
- $\nu(t)$ and $e(s)$ are independent for all t and s

Some fallacies with closed loop identification

- The closed loop experiment maybe non-informative even if the input is persistently exciting

- Spectral analysis applied in a straightforward fashion will give erroneous results

- Correlation analysis will give a biased estimate of the impulse response, since $u(t)$ and $H(q^{-1})e(t)$ are no longer uncorrelated

Example 1

Consider the first-order model structure

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$
$$u(t) = -fy(t)$$

The closed loop system appears to be

$$y(t) + (a + bf)y(t-1) = e(t)$$

We conclude that *all models* $(\hat{a}, \hat{b})$ subject to

$$\hat{a} = a + \gamma f, \quad \hat{b} = b - \gamma$$

with γ an arbitrary scalar, give the same IO description as the model (a, b)

The parameters are not consistently estimated, though $u(t)$ is persistently exciting

Example 2

This example will show that spectral analysis may not give identifiability

Closed loop behaviour: by omitting q^{-1}

$$y = [I + GF]^{-1}(GL\nu + He)$$
$$u = [L - F(I + GF)^{-1}GL]\nu - F(I + GF)^{-1}He$$

Assume that $L(q^{-1}) = 1$ and introduce the signal

$$z(t) = F(q^{-1})H(q^{-1})e(t)$$

We obtain the input and output in the following forms

$$y(t) = \frac{1}{1 + G(q^{-1})F(q^{-1})}\left[G(q^{-1})\nu(t) + \frac{1}{F(q^{-1})}z(t)\right]$$
$$u(t) = \frac{1}{1 + G(q^{-1})F(q^{-1})}[\nu(t) - z(t)]$$

Therefore, the spectral densities are given by

$$S_u(\omega) = \frac{1}{|1+G(\omega)F(\omega)|^2}[S_\nu(\omega) + S_z(\omega)]$$

$$S_{yu}(\omega) = \frac{1}{|1+G(\omega)F(\omega)|^2}\left[G(\omega)S_\nu(\omega) - \frac{1}{F(\omega)}S_z(\omega)\right]$$

The estimate of $G(\omega)$ is given by

$$\hat{G}(\omega) = \frac{S_{yu}(\omega)}{S_u(\omega)} = \frac{G(\omega)S_\nu(\omega) - S_z(\omega)/F(\omega)}{S_\nu(\omega) + S_z(\omega)}$$

- If there is no disturbances ($e = 0$) then $z(t) = 0$ and $\hat{G}$ is simplified to

$$\hat{G}(\omega) = G(\omega)$$

  It is possible to identify the true system dynamics

- If $\nu(t) = 0$

$$\hat{G}(\omega) = -\frac{1}{F(\omega)}$$

  The result is the negative inverse of the feedback

- In general case, the deviation of $\hat{G}$ from G is given by

$$\hat{G}(\omega) - G(\omega) = -\frac{1 + G(\omega)F(\omega)}{F(\omega)} \frac{S_z(\omega)}{S_\nu(\omega) + S_z(\omega)}$$

The spectral analysis applied in the usualy way gives *biased* estimates if there is a feedback acting on the system

Approaches to closed loop identification

Assume that PEM is used for parameter estimation for all approaches

1. **Direct approach**

   The existence of feedback is neglected and the recorded data are treated as if the system were operating in open loop, and not using the reference signal ν

2. **Indirect approach**

   Identify the closed loop system from the reference input ν to output y and retrieve from that the open loop system, making use of the known feedback law

3. **Joint input-output approach**

   Consider y and u as outputs of a system driven by ν (if measured) and noise. Recover the knowledge of the system and the regulator from this joint model

Direct identification

- a natural approach to closed loop data analysis
- works regardless of the complexity of the regulator
- requires no knowledge about the character of the feedback
- no special algorithms and software are required
- consistency and optimal accuracy are obtained if the model structure contains the true system
- unstable systems can be handled without problems as long as the closed loop is stable and the predictor is stable
- need good noise models

Model used for prediction:

$$y(t) = Gu(t) + H\nu(t), \quad \mathbf{E}\,\nu^2(t) = \lambda^2$$

Data used: $\{y(t), u(t)\}_{t=1}^{N}$

Goal: Estimate (SISO-case)

$$\begin{aligned} \hat{\theta} &= \underset{\theta}{\operatorname{argmin}}\, f(\theta) \\ f(\theta) &= \frac{1}{N}\sum_{t=1}^{N} e^2(t,\theta) \\ e(t,\theta) &= H^{-1}(y(t) - Gu(t)) \end{aligned}$$

Question: Identifiability ? desired solution:

$$\hat{G} = G, \quad \hat{H} = H$$

Consistency

Analyze the asymptotic cost function:

$$\bar{f}(\theta) = \lim_{N \to \infty} f(\theta) = \mathbf{E}\, e^2(t, \theta)$$

- System identifiable:

  Will $\hat{G} = G$ and $\hat{H} = H$ be a global minimum to $\bar{f}(\theta)$?

- Consistency:

  Is the solution $\hat{G} = G$ and $\hat{H} = H$ unique?

Example

Consider a first-order system

$$y(t) + ay(t-1) = bu(t-1) + \nu(t), \quad \mathbf{E}\,\nu(t)\nu(s) = \lambda^2\delta_{t,s}$$

The model structure is

$$y(t) + \hat{a}y(t-1) = \hat{b}u(t-1) + e(t)$$

The input is assumed to come from a time-varying proportional regulator:

$$u(t) = \begin{cases} -f_1 y(t), & \text{for a proportion } \gamma_1 \text{ of the total time} \\ -f_2 y(t), & \text{for a proportion } \gamma_2 \text{ of the total time} \end{cases}$$

Then the closed loop system is given by

$$y_i(t) + (a + bf_i)y_i(t-1) = \nu(t)$$

$$e_i(t) = y_i(t) + (\hat{a} + \hat{b}f_i)y_i(t-1)$$

The loss function is

$$\begin{aligned}\bar{f}(\hat{a},\hat{b}) &= \gamma_1\,\mathbf{E}\,e_1^2(t) + \gamma_2\,\mathbf{E}\,e_2^2(t)\\ &= \lambda^2 + \gamma_1\lambda^2\left(\frac{(\hat{a}+\hat{b}f_1-a-bf_1)^2}{1-(a+bf_1)^2}\right) + \gamma_2\lambda^2\left(\frac{(\hat{a}+\hat{b}f_2-a-bf_2)^2}{1-(a+bf_2)^2}\right)\end{aligned}$$

We can show that

$$\bar{f}(\hat{a},\hat{b}) \geq \lambda^2 = \bar{f}(a,b)$$

- We obtain that $\hat{a}=a$ and $\hat{b}=b$ is a global minimum
- Solve $\bar{f}(\hat{a},\hat{b}) = \lambda^2$

$$\begin{bmatrix}1 & f_1\\ 1 & f_2\end{bmatrix}\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix} = \begin{bmatrix}a+bf_1\\ a+bf_2\end{bmatrix}$$

- We obtain a unique solution *iff* $f_1 \neq f_2$
- The use of two different regulators is sufficient to give parameter identifiability

General case

- The desired solutions $\hat{H} = H$ and $\hat{G} = G$ will be a global minimum of $\bar{f}(\theta)$
- Unique global minimum is necessary for parameter identifiability (consistency)
- Consistency is assured by
  - Using an external input $\nu(t)$
  - Using a regulator $F(q^{-1})$ that shifts between different settings

Indirect identification

The closed loop system is given by

$$y(t) = G_c(q^{-1})\nu(t) + H_c(q^{-1})e(t)$$

$$G_c \triangleq (I + GF)^{-1}GL, \quad H_c \triangleq (I + GF)^{-1}H$$

The indirect approach contains two steps:

- Step 1: Identify the closed loop using $\nu(t)$ as input and $y(t)$ as output

  Estimate $\hat{G}_c$ and $\hat{H}_c$ from $y(t), \nu(t)$ using PEM

- Step 2: Determine the open loop from the estimated closed loop system

  Form the estimates $\hat{G}, \hat{H}$, using the knowledge of the feedback F, L

Joint input-output identification

Consider $y(t)$ and $u(t)$ as outputs from a multivariable system:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{11}(q^{-1};\theta) & \mathcal{H}_{12}(q^{-1};\theta) \\ \mathcal{H}_{12}(q^{-1};\theta) & \mathcal{H}_{22}(q^{-1};\theta) \end{bmatrix} \begin{bmatrix} e(t) \\ \nu(t) \end{bmatrix}$$

- Use innovation model: $z(t) = (y(t), u(t))^T$

$$z(t) = \mathcal{H}(q^{-1};\theta)\bar{e}(t), \quad \mathbf{E}\,\bar{e}(t)\bar{e}(s)^* = \Omega(\theta)\delta_{t,s}$$

- Use PEM to identify $\mathcal{H}$ and Ω where

$$\mathcal{H}^{-1} \text{ is asymptotically stable and } \mathcal{H}(0;\theta) = I$$

Example

Consider the system

$$y(t) + ay(t-1) = bu(t-1) + e(t) + ce(t-1)$$

where $|c| < 1$, $e(t)$ is white noise with $\mathbf{E}\, e^2(t) = \lambda^2$, and the feedback is

$$u(t) = -fy(t) + \nu(t)$$

where $\nu(t)$ is white noise with variance σ^2 and independent of $e(s)$

The closed loop system is given by

$$(1 + \alpha q^{-1})y(t) = bq^{-1}\nu(t) + (1 + cq^{-1})e(t)$$

where $\alpha = a + bf$ and we assume that $|\alpha| < 1$

We can write $z(t) = (y(t), u(t))^T$ as

$$z(t) = \frac{1}{1+\alpha q^{-1}} \begin{bmatrix} 1+cq^{-1} & bq^{-1} \\ -f(1+cq^{-1}) & 1+aq^{-1} \end{bmatrix} \begin{bmatrix} e(t) \\ \nu(t) \end{bmatrix}$$

- $\mathcal{H}^{-1}$ has a denominator containing $(1+cq^{-1})(1+\alpha q^{-1})$ which is stable
- However, $\mathcal{H}(0;\theta) = \begin{bmatrix} 1 & 0 \\ f & 1 \end{bmatrix}$

To obtain the innovation form, we can modify $z(t)$ as

$$\begin{aligned} z(t) &= \frac{1}{1+\alpha q^{-1}} \begin{bmatrix} 1+cq^{-1} & bq^{-1} \\ -f(1+cq^{-1}) & 1+aq^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ f & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -f & 1 \end{bmatrix} \begin{bmatrix} e(t) \\ \nu(t) \end{bmatrix} \\ &= \frac{1}{1+\alpha q^{-1}} \begin{bmatrix} 1+(c+bf)q^{-1} & bq^{-1} \\ f(a-c)q^{-1} & 1+aq^{-1} \end{bmatrix} \begin{bmatrix} e(t) \\ -fe(t)+\nu(t) \end{bmatrix} \end{aligned}$$

We can conclude that

$$\mathcal{H}(q^{-1};\theta) = \frac{1}{1+\alpha q^{-1}} \begin{bmatrix} 1+(c+bf)q^{-1} & bq^{-1} \\ f(a-c)q^{-1} & 1+aq^{-1} \end{bmatrix}$$

$$\Omega(\theta) = \mathbf{E} \begin{bmatrix} e(t) \\ -fe(t)+\nu(t) \end{bmatrix} \begin{bmatrix} e(t) \\ -fe(t)+\nu(t) \end{bmatrix}^* = \begin{bmatrix} \lambda^2 & -f\lambda^2 \\ -f\lambda^2 & f^2\lambda^2+\sigma^2 \end{bmatrix}$$

- Use PEM to fit a first-order ARMA model $\hat{\mathcal{H}}$ to $z(t)$
- From the consistency of PEM, we can identify the parameters from

$$\hat{\mathcal{H}} = \mathcal{H}, \quad \hat{\Omega} = \Omega$$

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