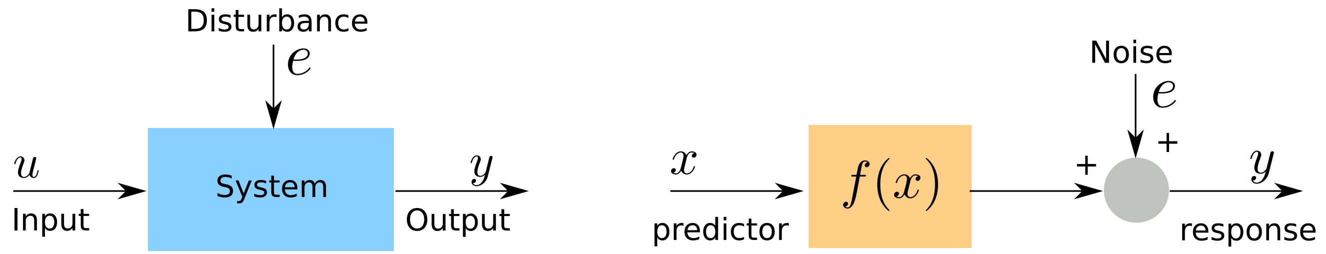
1. Introduction

- basic concept
- system identification methods
- procedures in system identification
- examples

Basic concept

objective: how to build a system description from experimental data



when we talk about a model

- a dynamical model with input u and output y: $y = Gu$
- a statistical model with predictor x and response y: $y = f(x)$

due to uncertainty of measurement or unexplained phenomenon

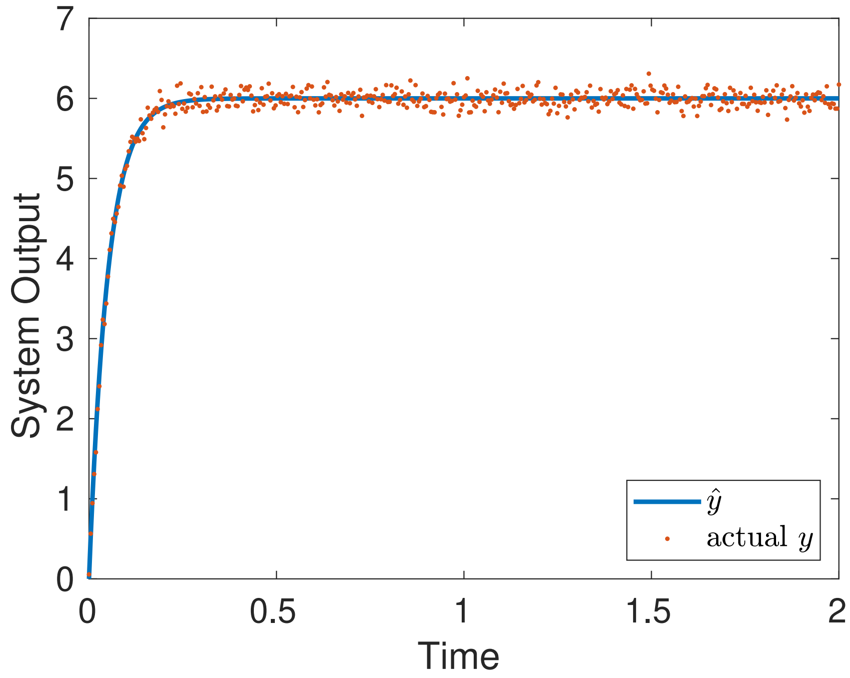
the output is assumed to be corrupted by noise

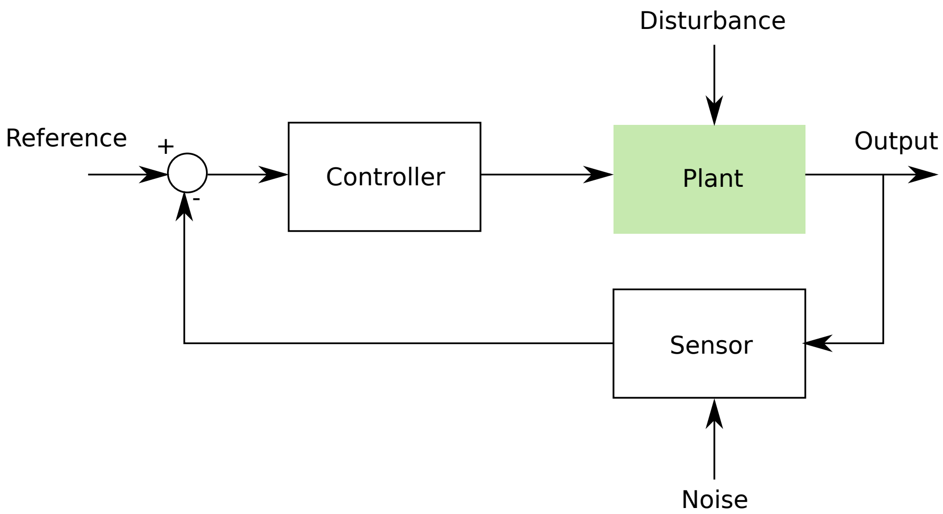
Model usages

estimation of system description can serve for many purposes:

- obtain a mathematical model for controller design
- explain/understand observed phenomena (e.g., machine learning)
- forecast events in the future (e.g., time series analysis in econometrics)
- obtain a model of signal in filter design (e.g., signal processing)
- model inference

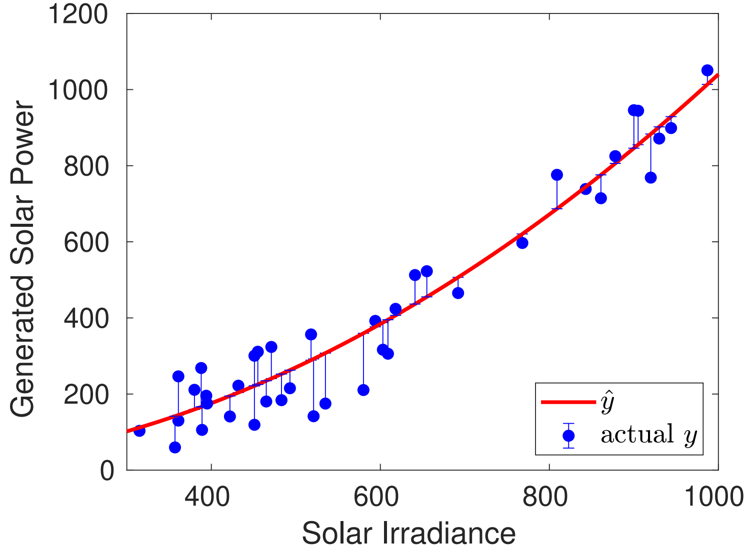
System Identification for controller design

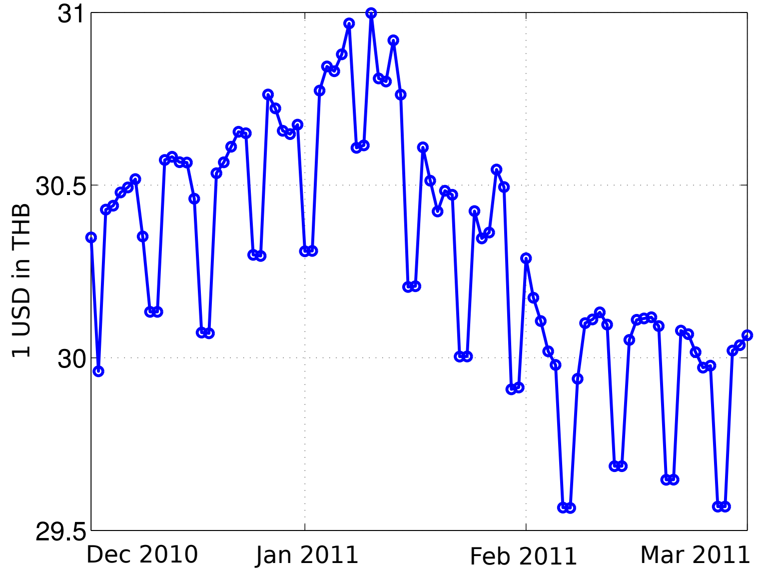




- for controller design, the plant is assumed known
- in system identification, we aim to estimate the parameters in a model

System Identification for prediction





- left: estimate generated solar power from measurements of solar irradiance

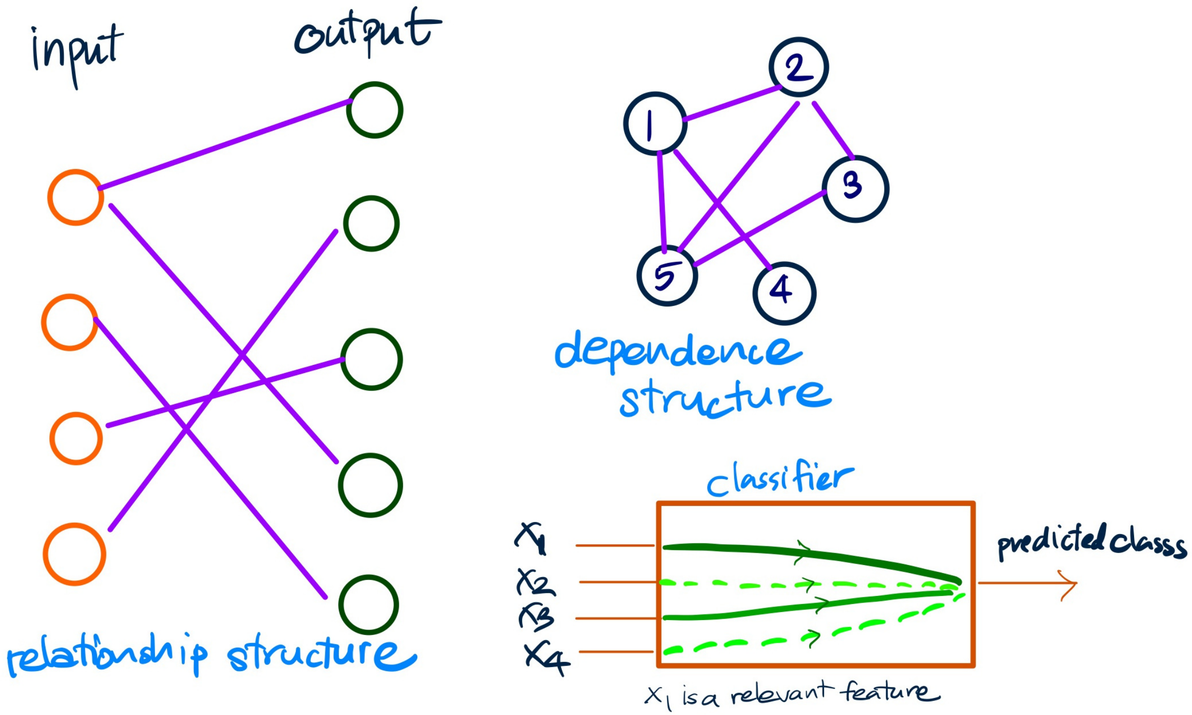
$$\text{solar power} = f(\text{solar irradiance}) \approx \beta_0 + \beta_1 I + \beta_2 I^2 + \cdots + \beta_n I^n$$

- right: forecast the Thai Baht in Apr, May,... ? need a model for prediction

$$\hat{x}_{\text{Apr}} = a_1 x_{\text{Mar}} + a_2 x_{\text{Feb}}$$

Model inference

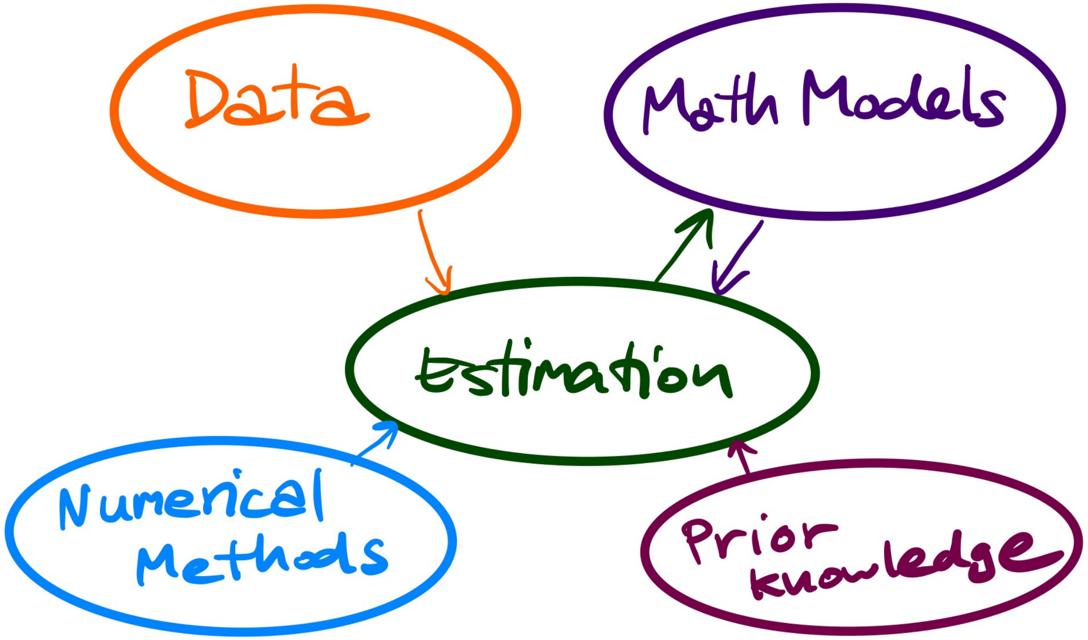
model parameters (or its function) can *infer* some pattern of data



- interconnection structure between (y, u) or among the variables
- relevancy of using a set of features to explain the response variables

Essential elements

users develop a math model to explain data using prior knowledge of applications



- applicable estimation techniques depend on a selected model
- most model estimation problems require numerical methods to get a numerical solution

Models

a description of the system, or a relationshop among observed signals

a model should capture the essential information about the system

types of models

- graph and tables, e.g., bode plots and step response
- mathematical models, e.g., differential and difference equations
- probablilistic models, e.g, probability density function

System identification is a process of obtaining models based on a data set collected from experiments

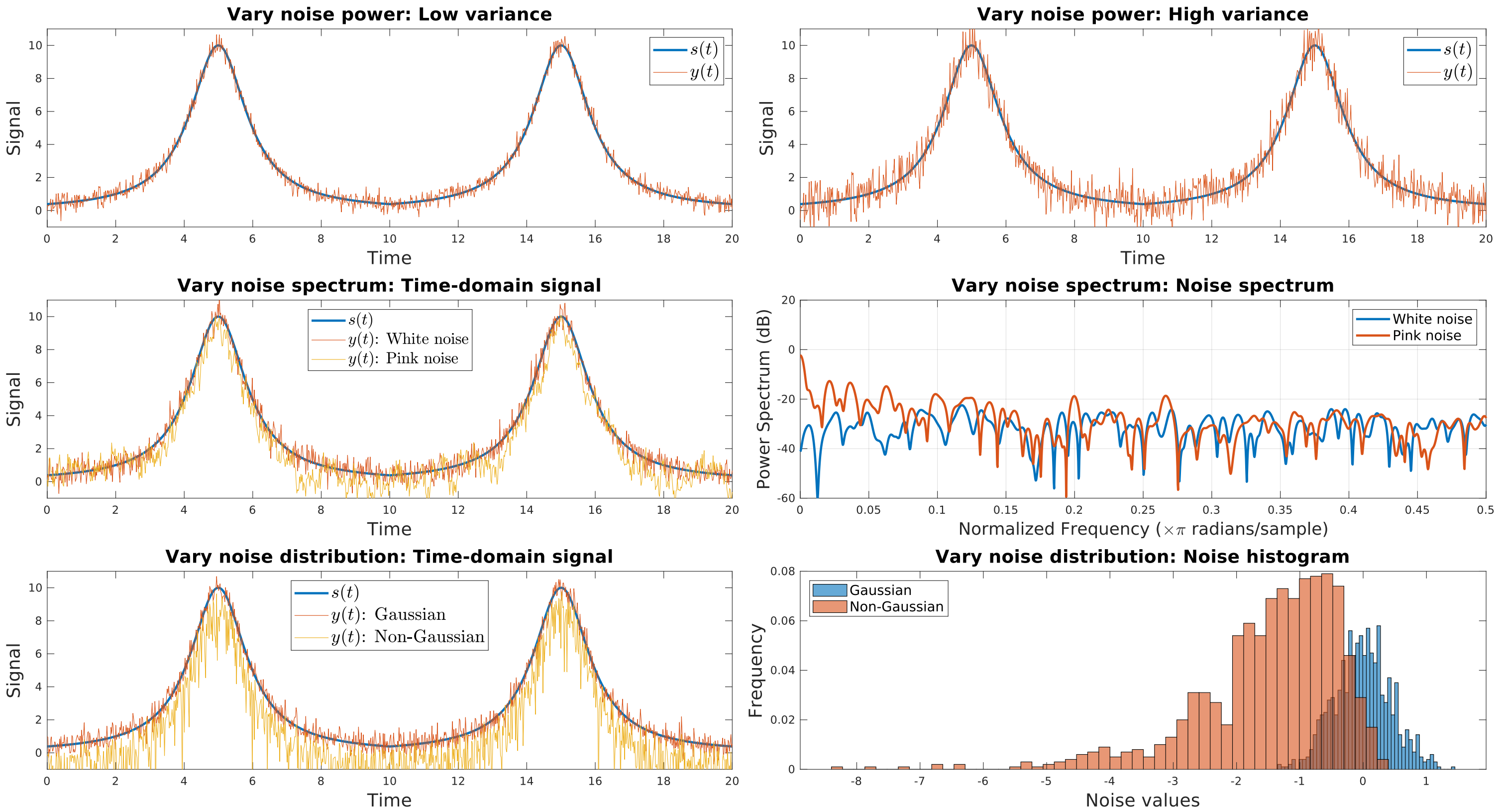
input and output signals from the system are recorded and analyzed to infer a model

System identification methods

- **Nonparametric approach**
  - aim at determining a (time/frequency) response directly without first selecting a possible set of models
  - gives basic information about the sytsem and is useful for validation
  - examples are transient analysis, frequency analysis, correlation analysis, and spectral analysis
- **Parametric approach**
  - require assumptions on a model class/structure
  - the search for the best model within the candidate set becomes a problem of determining the model parameters
  - typically more complicated than the nonparametric approach
  - results can be further used for controller design, simulation, etc.

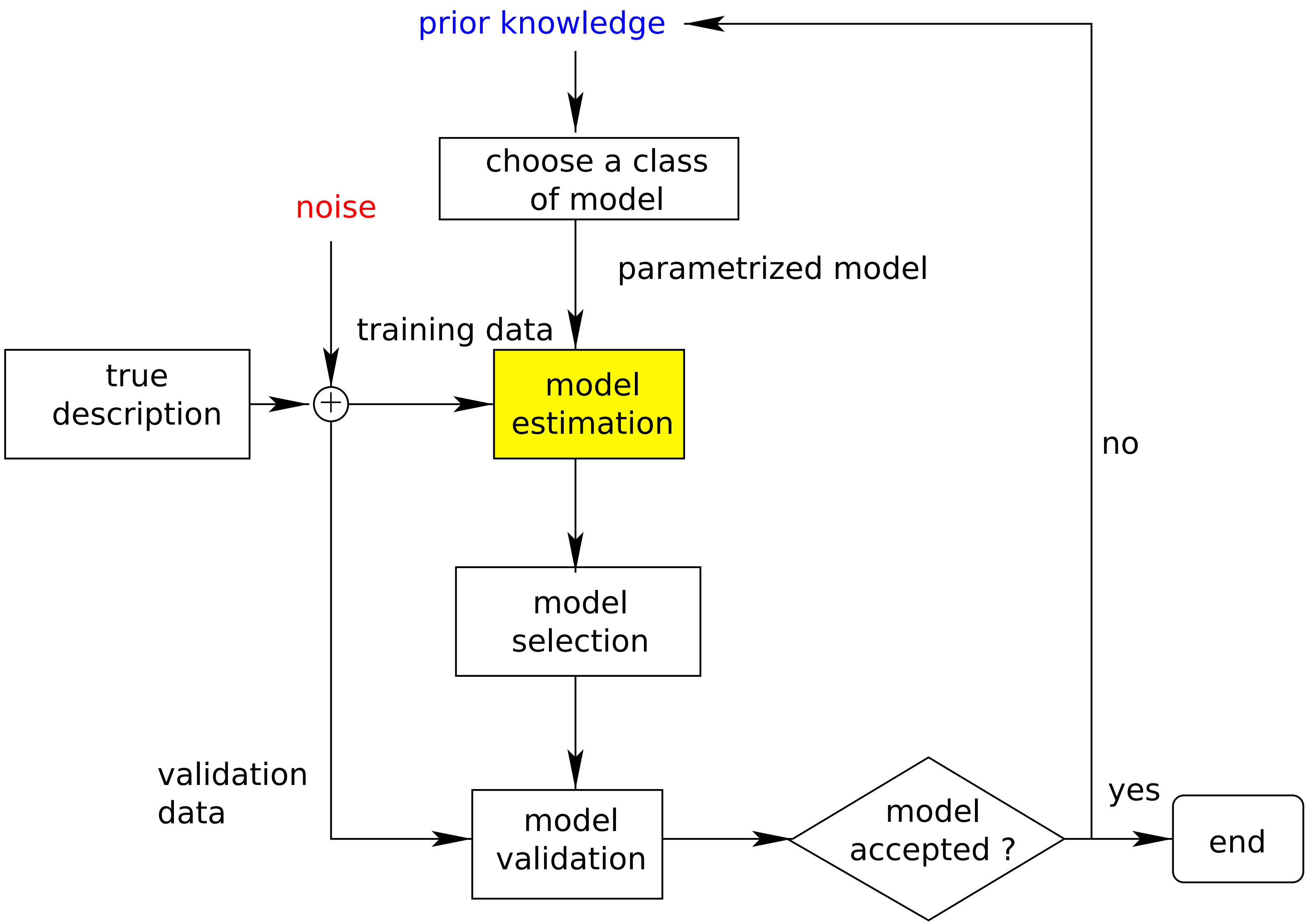
Prior knowledge about uncertainty

a model is *estimated* in some optimal sense to find a best match to data



noise characteristic is one prior assumption that users can exploit in estimation

Procedures in System Identification



Parametric estimation

- model class:

  SISO/MIMO, linear/nonlinear, time-invariant/time varying, discrete/continuous

- searching the best model within a candidate set becomes a problem of determining the model parameters

- the selected parameter $\hat{x}$ from a model class $\mathcal{M}$ is optimal in some sense, i.e.,

$$\hat{x} = \underset{x \in \mathcal{M}}{\operatorname{argmin}} \, f(x, \mathcal{D}),$$

  where f is a measure of goodness of fit (or loss function) and is a function of information data ($\mathcal{D}$)

- examples of f are quadratic loss, likelihood, entropy function, etc.

Estimation methods

- linear least-squares method (LS)

  simple to compute, no assumption on noise model

- statistical estimation methods, e.g., maximum likelihood, Bayes

  use prior knowledge about noise

- instrumental-variable method

  a modification of the LS method for correlated noise

- subspace methods

  LS and projection framework of estimating state-space models

- prediction-error method

  model the noise, applicable to a broad range of models

Model selection

- **Principle of parsimony:**

  one should pick a model with the smallest possible number of parameters that can adequately explain the data

- one can trade off between

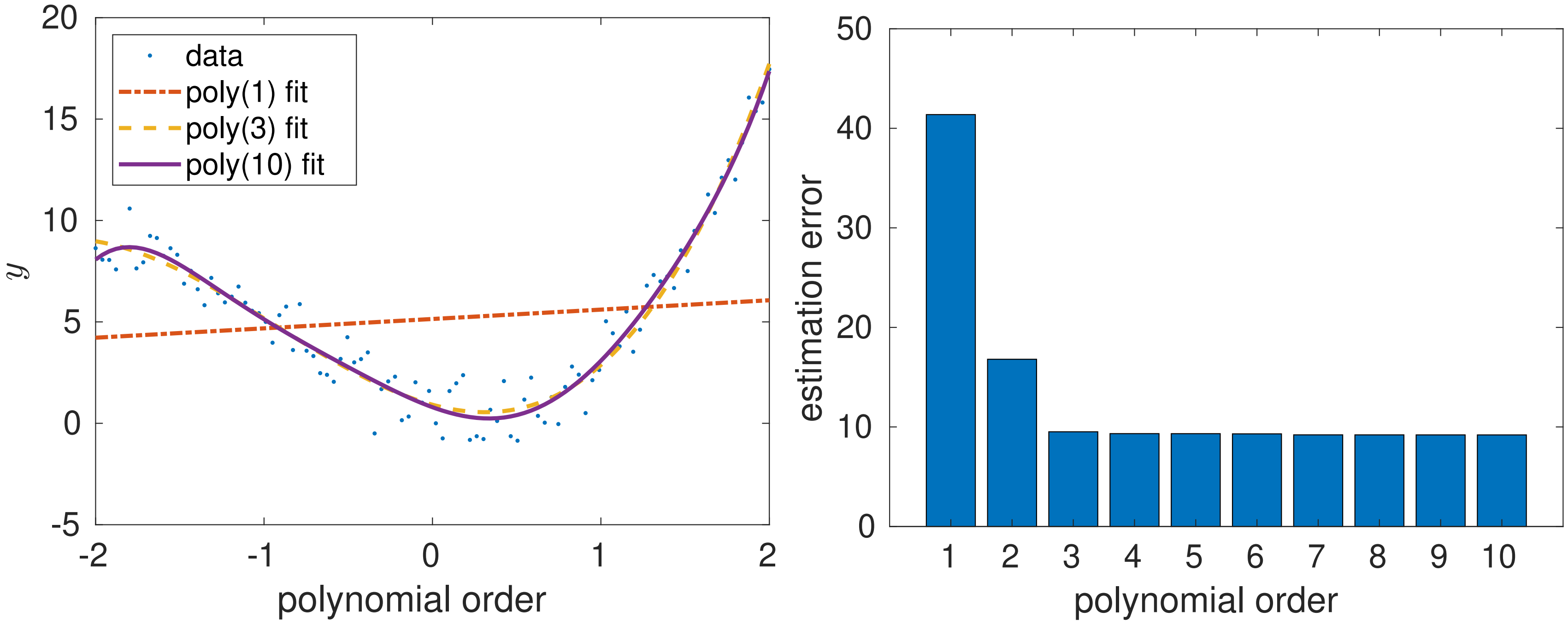
Goodness of fit vs **Complexity**

- related to the concept of bias VS variance in statistics

- examples of model selection criterions are FPE, AIC, BIC, etc.

Example: Polynomial fitting

curve fitting problem of polynomial of order n (true order is $n = 3$)

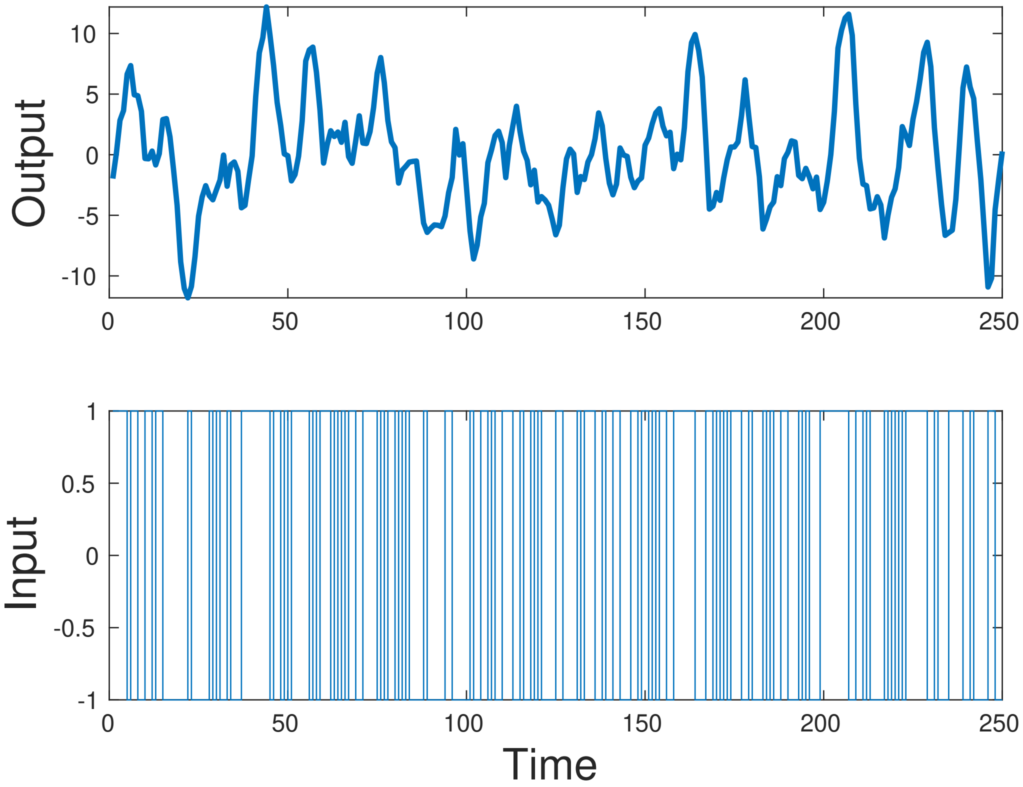


- the error begins to decrease as the model picks up the relevant features
- as the model order increases, the model tends to *over fit* the data

Model validation

- a parametric estimation procedure picks out the *best* model
- a problem of model validation is to verify whether this best model is "good enough"
- test the estimated model (obtained from training data), with a new set of data (validation set)
- the tests verify whether the dynamic from the input and the noise model are adequate

Numerical Example



- feed a known input to the system and measure the output
- the input should contain rich information to excite the system

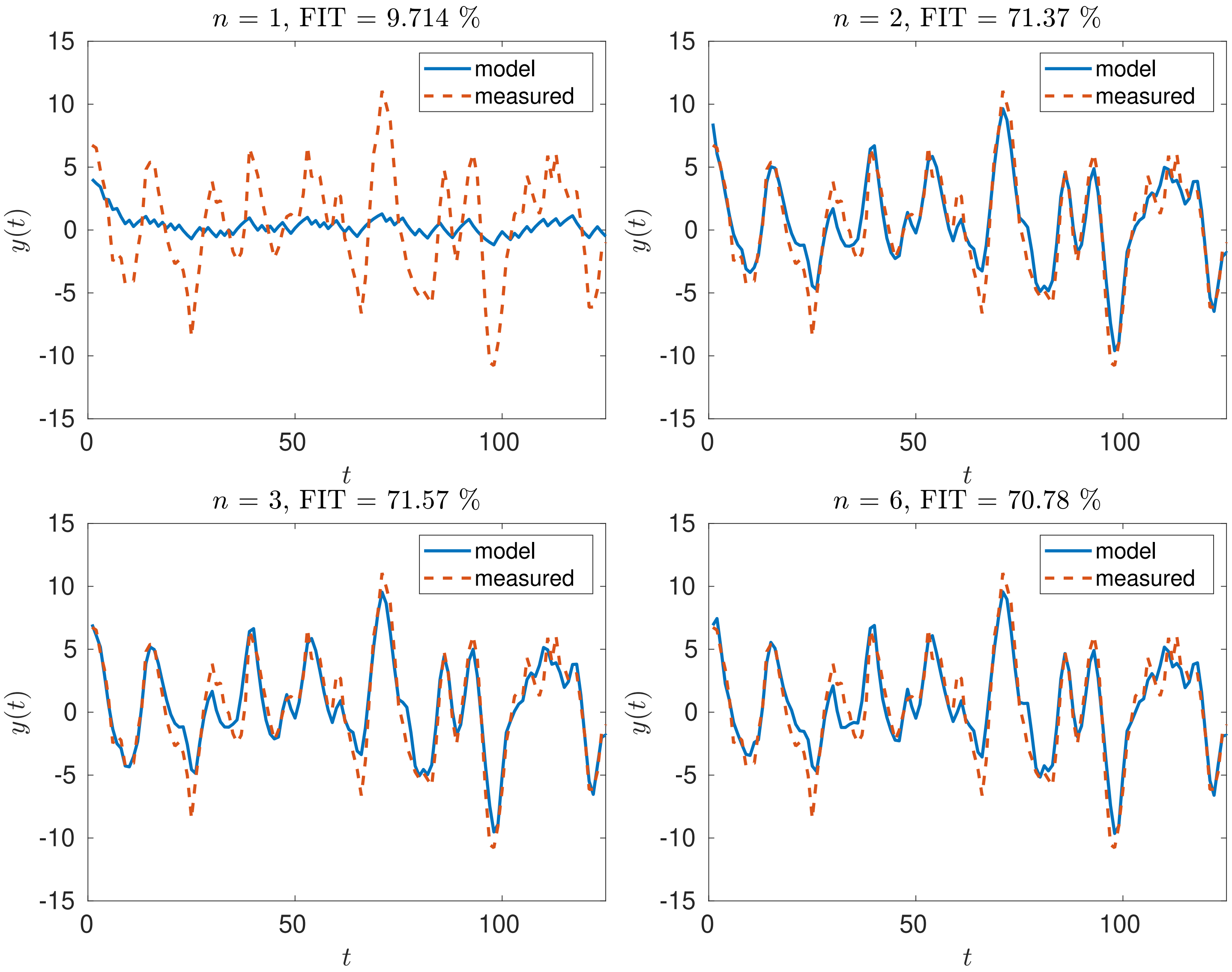
- fit the measured output to the model

$$(1 + a_1 q^{-1} + \cdots + a_n q^{-n}) y(t) = \\ (b_1 q^{-1} + \cdots + b_n q^{-n}) u(t) + (1 + c_1 q^{-1} + \cdots + c_n q^{-n}) e(t)$$

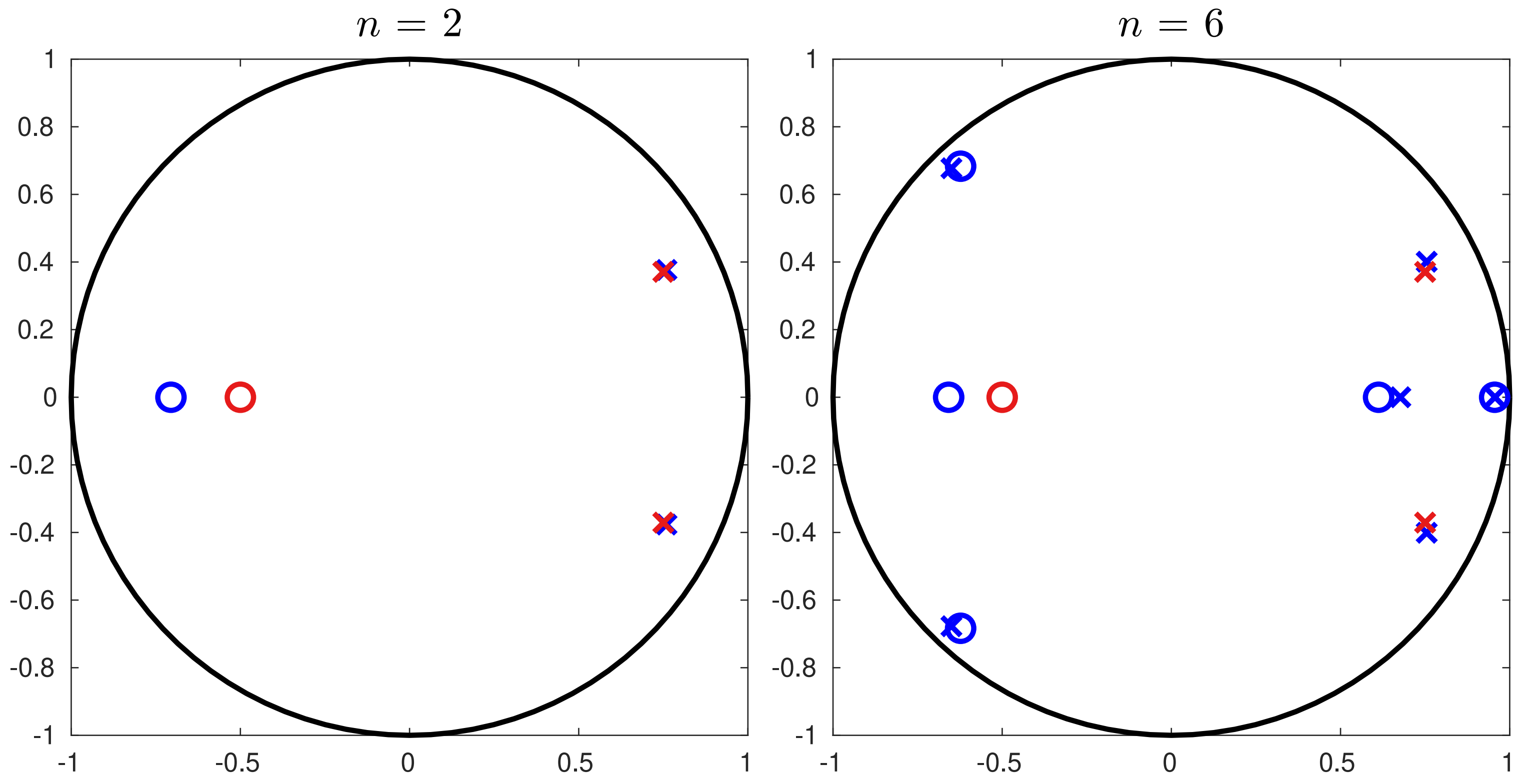
  with unknown parameters $a_1, \ldots, a_n, b_1, \ldots, b_n, c_1, \ldots, c_n$

- this model is known as **Autoregressive Moving Average with Exogenous input (ARMAX)**
- $e(t)$ represents the noise that enters to the system
- n is the model order, which is selected via *model selection*
- the parameters are estimated by the *prediction-error method (PEM)*

Example of output prediction



Example of zero-pole location



- ○: zeros, ×: poles
- red: true system, blue: estimated models
- chance of zero-pole cancellation at higher order

Skills needed for system identification

one should have

- concepts of dynamical systems (description, how to analyze their properties)
- probability and statistics (to understand probablilistic models, estimation methods, to statistically interpret results)
- linear algebra (many linear models involve matrix analysis)
- optimization (most model estimations are optimization problems)
- programming (for numerical methods to solve estimation problems)

References

Chapter 1,2 in
L. Ljung, *System Identification: Theory for the User*, Prentice Hall, Second edition, 1999

Chapter 1-3 in
T. Söderström and P. Stoica, *System Identification*, Prentice Hall, 1989

L. Ljung, *Perspective on System Identification*,
`http://www.control.isy.liu.se/~ljung/`