2102531 System Identification

Term Project
Semester 1/2017
Recursive Estimation of Solar Irradiance using Time-Series Model

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Abstract

In generally, when the new data is measured. The estimated model are changed but estimate by the data which combine between both historical data and new data. That meant estiamte every time when the new data is measured. This project is aim to describe updating an estimated model of solar forecasting by using recursive method with SARIMA model which is a type of time series model. The result from this study is a modification of the model which use recursive identification.

1 Introduction

Solar irradiance is the power per unit area from the sun and is one of the important factor for photovoltaic cell to make a renewable energy from solar radiation. In Meteorological Department has separate the solar irradiance into 3 types

1. Global Horizontal Irradiation(GHI) is the total of Direct Normal Irradiation(DNI) and Diffuse Horizontal Irradiance(DHI). GHI is an important parameter for evaluation of solar energy potential of a particular region.

2. Direct Normal Irradiation(DNI) is the amount of solar radiation received per unit area by a surface that is always held perpendicular (or normal) to the rays that come in a straight line from the direction of the sun at its current position in the sky.

3. Diffuse Horizontal Irradiance(DHI) is the amount of radiation received per unit area by a surface that does not arrive on a direct path from the sun, but has been scattered by molecules and particles in the atmosphere

All three component can write to equation 1

$$GHI = DHI + DNI \cos(\theta) \tag{1}$$

where θ is solar zenith angle. This study is focus on GHI only. The behavior of the solar irradiance in two days is shown in figure 1. This figure show that the GHI data is similar to a periodic signal.

This study focus on time series models. Time series model is a model which can analyzed to understand a past and forecast a future. This model are use in many branch such as engineering, account, and science where the data is measured sequentially in time. Time series split into 2 types

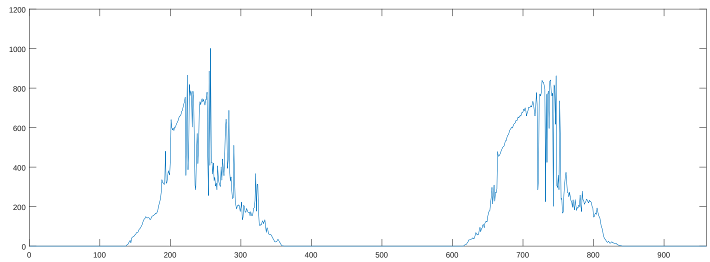


Figure 1: GHI in 1st-2nd January 2017

1. Time series model with stationary process is a time series model which have a constant mean and autocorrelation depend on time gap example AR, MA, ARMA, ARX, ARMAX
2. Time series model with non-stationary process is a time series model which either mean or autocorrelation depend on time example ARIMA, ARIMAX

From above show that the time series model with stationary process is a subset of time series model with non-stationary model. Thus, we will use time series model with non-stationary process for describe the trend from the past and forecast the future. According to [3] show that the $SARIMA(2,2,4)(0,1,1)_{16}$ have the most log-likelihood function. Thus, this study use the Seasonal AutoRegressive Integrated Moving Average (SARIMA) model which define by

$$\tilde{A}(L)(1-L^T)^D A(L)(1-L)^d I(t) = \tilde{C}(L)C(L)e(t) \tag{2}$$

can be written as $SARIMA(p,d,q)(P,D,Q)_T$ where

$$\begin{aligned} A(L) =& 1-(A_1L + A_2L^2 + \cdots + A_pL^p) \\ C(L) =& 1+(C_1L + C_2L^2 + \cdots + C_qL^q) \\ \tilde{A}(L) =& 1-(\tilde{A}_1L^T + \tilde{A}_2L^{2T} + \cdots + \tilde{A}_PL^{PT}) \\ \tilde{C}(L) =& 1+(\tilde{C}_1L^T + \tilde{C}_2L^{2T} + \cdots + \tilde{C}_QL^{QT}) \end{aligned} \tag{3}$$

L is a lag operator, T is a seasonal period, d is an integrated non-seasonal order and D is an integrated seasonal order. Each polynomial in equation 3 call

- $A(L)$ call autoregressive(AR) polynomial
- $C(L)$ call moving average(MA) polynomial
- $\tilde{A}(L)$ call seasonal autoregressive(SAR) polynomial
- $\tilde{C}(L)$ call seasonal moving average(SMA) polynomial

Equation 2 can reduced to the AutoRegressive Integrated Moving Average(ARIMA) model which define by

$$A(L)(1-L)^d \tilde{I}(t) = C(L)v(t) \tag{4}$$

can be written as $ARIMA(p,d,q)$ where

$$\begin{aligned} W(t) =& \tilde{A}(L)(1-L^T)^D I(t) \\ v(t) =& \tilde{C}(L)e(t) \end{aligned} \tag{5}$$

Equation 2 can reduced to the AutoRegressive Moving Average(ARMA) model which define by

$$A(L)y(t) = C(L)v(t) \tag{6}$$

can be written as $ARMA(p,q)$ where

$$y(t) \quad = \tilde{A}(L)(1-L^T)^D(1-L)^d I(t) \tag{7}$$

2 Problem statement

The objective of the problem statement are

- Using SARIMA model to forecast solar irradiance
- $I(t)$ is measured solar irradiance
- $\tilde{A}_1, \tilde{C}_2, \cdots, \tilde{A}_P$ given to zero
- $A_1, A_2, \cdots, A_p, C_1, C_2, \cdots, C_q, \tilde{C}_1, \tilde{C}_2, \cdots, \tilde{C}_Q$ are the coefficient which we want to find

3 Methodology

This study using time series model to estimate the solar irradiance which the process of the estimation is shown in figure 3.

First, the GHI data must be remove a seasonal trend. That meant we could write the seasonal

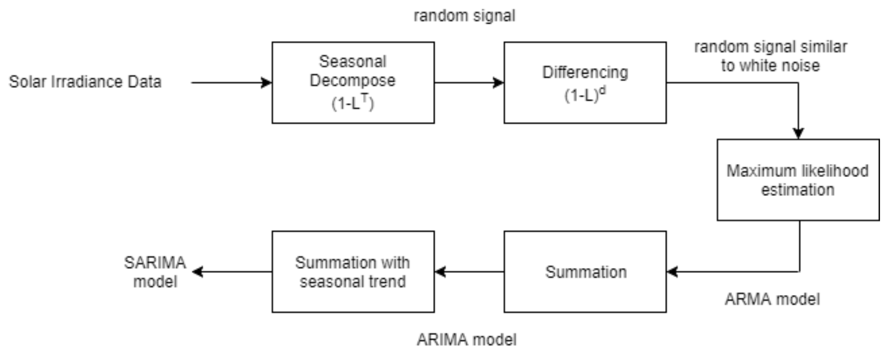


Figure 2: Estimation Diagram

trend into fourier series. After removing the seasonal trend, the removed seasonal trend data will similar to random signal. Before finding the parameter in ARMA model, we check the autocorrelation graph to check for sure that each point of data are uncorrelated. If the autocorrelation graph is not similar to white noise, we could differentiate that data and check the autocorrelation again. After differencing until the autocorrelation graph is similar to white noise, we use that set of differencing data to find the parameter in ARMA model by using maximum likelihood estimation(ML). Then this model use prediction error method(PEM) to find the estimated model and use this model to forecast GHI data.

3.1 Estimation of seasonal trend

This section is described finding the seasonal trend and use this seasonal trend to remove from the data. The aim of this section is finding the seasonal period T.

1. SARIMA Model
   This study consider an additive seasonal trend which can write in function 8

$$A(L)y(t) = s(t) + \alpha + C(L)e(t) \tag{8}$$

   where $s(t) = s(t-T)$ is a seasonal term, α is constant and $e(t)$ is noise. Then $y(t)$ is subtracted by $y(t-T)$. We will get

$$\begin{aligned} A(L)y(t) - A(L)y(t-T) &= s(t) + \alpha + C(L)e(t) - s(t-T) - \alpha - C(L)e(t-T) \\ &= C(L)e(t) - C(L)e(t-T) \end{aligned} \tag{9}$$

   The above model can write to $SARIMA(p,d,q)(0,1,1)_T$

2. Fitting seasonal trend
   This section is describe finding the seasonal ternd before removing from the data. The seasonal trend can write into fourier series

$$s(t) = \sum_{i=1}^{k}(a_i \cos(\omega_i t) + b_i \sin(\omega_i t)) \tag{10}$$

   where $t = 1, 2, \cdots, N$, a_i is coefficient of co-sinusoidal component of each frequency ω_i and b_i is coefficient of sinusoidal component of each frequency ω_i. To find the frequency ω_i, we must analytic in frequency domain to find the power spectrum density. Then we choose the high energy frequency to be selected as ω_i. Before finding the power spectrum density, we use Fast Fourier Transform(FFT) to be transformed to the frequency domain. Fast fourier transform is algorithm to find the discrete fourier transform(DFT) [2] which shown in equation 11

$$S(k) = \sum_{t=0}^{N-1} s(t) e^{\frac{j2\pi kt}{N}} \tag{11}$$

   where $\omega_k = \frac{2\pi k}{N}$ and $k = 0, 1, \cdots, N-1$. After using FFT, then we find $|S(k)|$ to find ω_k. Only high-energy frequency will given to ω_i in equation 10.

3.2 Estimation of integrated part

This section is described a process to find the integrated order. After removing seasonal trend, ACF might be correlated in each time. That meant ACF is not similar to white noise spectrum. From equation 6 we can write

$$A(L)y(t) = C(L)v(t)$$

The data $y(t)$ was differentiate by subtracting $y(t-1)$.

$$\Delta y(t) = y(t) - y(t-1) = (1-L)y(t)$$

We can also differentiate 2$^{\text{nd}}$ time

$$\Delta^2 y(t) = \Delta y(t) - \Delta y(t-1) = (1-L)^2 y(t)$$

If we differentiate the data in d time, we will write in equation

$$\Delta^d y(t) = (1-L)^d y(t) \tag{12}$$

Finally, the equation 6 is substituted $y(t)$ by equation 12. We will get the AutoRegressive Integrated Moving Average(ARIMA) Model

$$A(L)(1-L)^d y(t) = C(L)v(t) \tag{13}$$

From the equation 13, we will see that the transfer function is

$$\frac{Y(z)}{V(z)} = \frac{C(z)}{A(z)(1-z^{-1})^d} \tag{14}$$

In equation 14 show that this model has not stationary process because it has poles lying on unit circle. If we substitute $\hat{W}(t) = (1-L)^d y(t)$ this model will have the stationary process.

3.3 Estimation of ARMA Model

This section is describe about the estimation of AutoRegressive Moving Average(ARMA) model. In this report we will use maximum likelihood estimation to find the parameter in AR polynomial, MA polynomial and noise variance.
Maximum likelihood estimation is one of the method to find the parameter by maximizing a cost function which define by

$$\mathcal{L}(y|\theta) = f(y(1), y(2), \cdots, y(N)|\theta) \tag{15}$$

where $f(y(1), y(2), \cdots, y(N)|\theta)$ is defined by

$$f(y(1), y(2), \cdots, y(N)|\theta) = \prod_{t=1}^{N} \Big(\frac{1}{\sigma\sqrt{2\pi}}\Big) e^{-\frac{v(t)^2}{2\sigma^2}} \tag{16}$$

In equation 15, this cost function also called likelihood function where

$$\theta = \begin{bmatrix} A_1 & A_2 & \cdots & A_p & C_1 & C_2 & \cdots & C_q & \sigma^2 \end{bmatrix}^T$$

$f(y|\theta)$ is conditional probabilitiy density function(conditional pdf) of $v(t)$ in equation 2 and N is a number of data.
From equation 6 can also write

$$y(t) = A_1 y(t-1) + A_2 y(t-2) + \cdots + A_p y(t-p) + v(t) + C_1 v(t-1) + C_2 v(t-2) + \cdots + C_q v(t-q) \tag{17}$$

Thus, we can find $v(t)$ in term of $A_1, A_2, \cdots, A_p, C_1, C_2, \cdots, C_q$ from equation 17

$$v(t) = y(t) - (A_1 y(t-1) + A_2 y(t-2) + \cdots + A_p y(t-p)) - (C_1 v(t-1) + C_2 v(t-2) + \cdots + C_q v(t-q)) \tag{18}$$

If $v(t)$ has normal distribution which have zero mean and variance σ^2. Then the log-likelihood function according to [1] is

$$\mathcal{L}(\theta) = -\frac{N}{2}\log(2\pi) - \frac{N}{2}\log(\sigma^2) - \sum_{t=1}^{N} \frac{v(t)^2}{2\sigma^2} \tag{19}$$

From [1], if we have $y(t)$ has real value from 1 to p and $v(t) = 0$ since $t = p, p-1, \cdots, p-q+1$, so that $y(t)$ also has normal distribution. Thus, we start at $t = p+1$. At the same time, the conditional likelihood function is change to equation 20

$$\mathcal{L}(y|\theta) = f(y(p+1), y(p+2), \cdots, y(N)|y(1), y(2), \cdots, y(p), \theta) \tag{20}$$

Thus, the likelihood function is

$$\mathcal{L}(y|\theta) = (\frac{1}{2\pi\sigma^2})^{N-p} e^{-\sum_{t=p+1}^{N} \frac{v(t)^2}{2\sigma^2}} \tag{21}$$

From the maximum likelihood estimati on method, we can find the estimator from log-likelihood function. Finally, we have a cost function to find the estimator from the maximum of the cost function in equation 22

$$\log \mathcal{L}(y|\theta) = -\frac{N-p}{2}\log(2\pi) - \frac{N-p}{2}\log(\sigma^2) - \sum_{t=p+1}^{N} \frac{v(t)^2}{2\sigma^2} \tag{22}$$

where $\log \mathcal{L}(y|\theta)$ is the cost function of the problem and $v(t)$ in equation 22 can consider in 2-norm. Thus we can write into matrix form

$$\begin{bmatrix} v(p+1) \\ v(p+2) \\ \vdots \\ v(N) \end{bmatrix} =$$

$$\begin{bmatrix} y(p+1) \\ y(p+2) \\ \vdots \\ y(N) \end{bmatrix} - \begin{bmatrix} y(p) & y(p-1) & \cdots & y(1) & v(p) & v(p-1) & \cdots & v(p-q+1) \\ y(p+1) & y(p) & \cdots & y(2) & v(p+1) & v(p) & \cdots & v(p-q+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ y(N) & y(N-1) & \cdots & y(p) & v(N) & v(N-1) & \cdots & v(p) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \\ C_1 \\ C_2 \\ \vdots \\ C_q \end{bmatrix}$$

The numerical solution of $\hat{\theta}_{ML}$ can find from many optimization method example steepest-descent, quasi-newton, conjugate-gradient and the other method. However, doing ML in MATLAB and doing ML by hand-out probably not given the same parameter especially the parameter in seasonal moving average polynomial because the simple estimation in equation 9 show that they has pole lie in unit circle.

3.4 Model Validation

After we specify some properties of the models. We must have some criterior score to find the optimum order of SARIMA models. This study using akaike information criterion(AIC) and bayesian information criterion(BIC). Both AIC and BIC are explain a trade-off between a complexity of the model and goodness of fit. Both AIC and BIC is defined by

$$AIC = -2\mathcal{L} + 2d \tag{23}$$

$$BIC = -2\mathcal{L} + d\log(N) \tag{24}$$

where $\mathcal{L}$ is log-likelihood function, N is number of training data and d is number of parameter in each models.

3.5 Computation of Forecast

After finding the model, it will be used to forecast the solar irradiance in the next h-step. From the ARMA model in equation 6.

$$A(L)y(t) = C(L)v(t)$$

The optimal prediction from the ARMA model by using prediction error method(PEM) in [4] and [5] is

$$\hat{y}(t|t-1) = (1 - C^{-1}(L)A(L))y(t) \tag{25a}$$

$$e(t) = C^{-1}(L)A(L)y(t) \tag{25b}$$

We can find the estimatied $ARMA(p,q)$ model

$$\begin{aligned} C(L)\hat{y}(t|t-1) &= (C(L) - A(L))y(t) \\ (C(L)-1)\hat{y}(t|t-1) + \hat{y}(t|t-1) &= (C(L)-1)y(t) - (A(L)-1)y(t) \\ \hat{y}(t|t-1) &= (C(L)-1)(y(t) - \hat{y}(t|t-1)) - (A(L)-1)y(t) \\ \hat{y}(t|t-1) &= (C(L)-1)e(t) - (A(L)-1)y(t) \\ \hat{y}(t|t-1) &= (C_1L + C_2L^2 + \cdots + C_qL^q)e(t) + (A_1L + A_2L^2 + \cdots + A_pL^p)y(t) \end{aligned} \tag{26}$$

So that we can compute one-step ahead prediction of $ARMA(p,q)$ model

$$\begin{aligned}\hat{y}(t+1|t) =&(C(L)-1)e(t+1)+(1-A(L))\hat{y}(t+1|t)\\ =&(C_1L+C_2L^2+\cdots+C_qL^q)e(t+1)+(A_1L+A_2L^2+\cdots+A_pL^p)\hat{y}(t+1|t)\\ =&C_1e(t)+C_2e(t-1)+C_3e(t-2)+\cdots+C_qe(t-q+1)+\\ &A_1\hat{y}(t|t)+A_2\hat{y}(t-1|t)+\cdots+A_p\hat{y}(t-p+1|t)\end{aligned} \tag{27}$$

And we can compute h-step prediction of $ARMA(p,q)$ model

$$\begin{aligned}\hat{y}(t+h|t) =&(C(L)-1)e(t+h)+(1-A(L))\hat{y}(t+h|t)\\ \hat{y}(t+h|t) =&C_1e(t+h-1)+C_2e(t+h-2)+C_3e(t+h-3)+\cdots+C_qe(t+h-q)+\\ &A_1\hat{y}(t+h-1|t)+A_2\hat{y}(t+h-2|t)+\cdots+A_p\hat{y}(t+h-p|t)\end{aligned} \tag{28}$$

where

$$\hat{y}(t+h|t)=\begin{cases}\hat{y}(t+h|t) & t>0\\ y(t+h) & t\leq 0\end{cases} \tag{29}$$

$$e(t+h|t)=\begin{cases}0 & t>0\\ e(t+h) & t\leq 0\end{cases} \tag{30}$$

MATLAB have command to forecast the h-step prediction after estimated the model. There are

1. Infer
   Infer gives the residual error and conditional variance from the data which we use. Then the fitted numerical value can find from the different between the data which use in this command and residual error.

2. Forecast
   Forecast give the predicted value from the estimated model and the data.

4 Experiments

In this section we used solar irradiance data since January 2017 to May 2017 for model estimation and solar irradiance data at June 2017 for model validation. This data set has sampling rate 3 minutes.

4.1 Seasonal Decompose

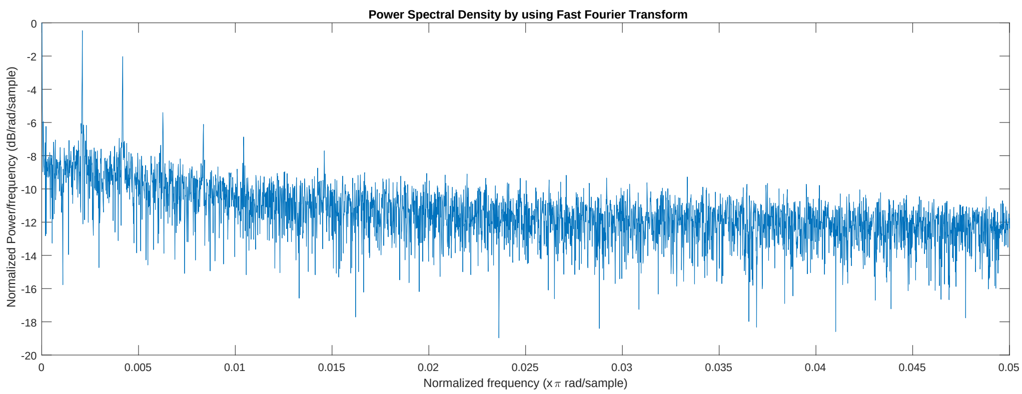


Figure 3: Power Spectral Density by using Fast Fourier Transform

First, we use fast fourier transform finding the power spectrum density. Then we choose only high energy frequency and give them ω_i. Figure 4.1 show that the solar irradiance data has 5 peaks frequency at $\omega_1 = 0.002\pi$, $\omega_2 = 0.004\pi$, $\omega_3 = 0.006\pi$, $\omega_4 = 0.008\pi$,and $\omega_5 = 0.01\pi$
After choosing frequency ω_i, we can find each a_i, b_i and α in equation 8 and 10 from the least square method. Then we will get the seasonal trend in figure 4.1.
Finally, we get the data with removing seasonal trend in figure 4.1.

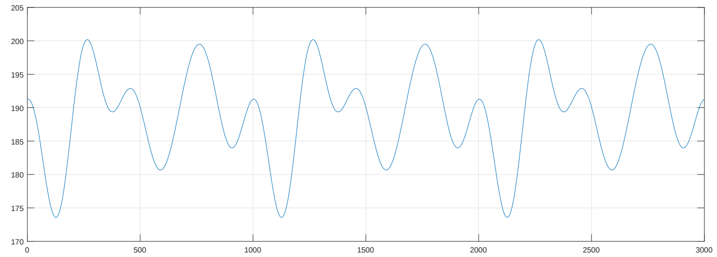


Figure 4: Seasonal trend of training data set

Figure 4.1 show that there is the residual error between the data and the fitting seasonal trend.

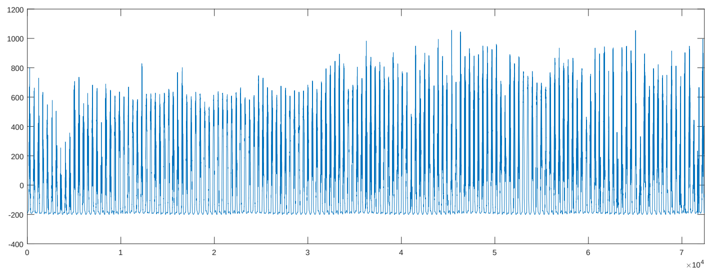
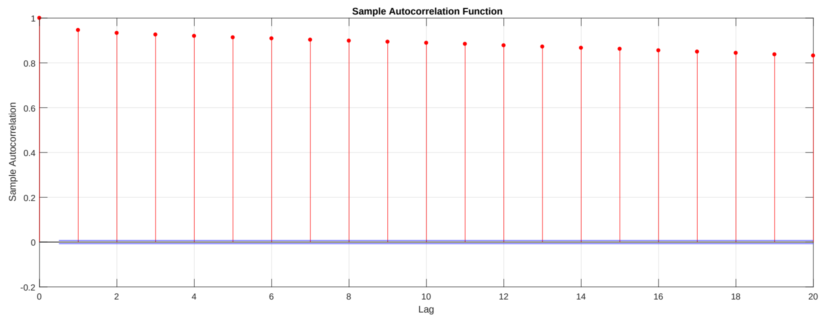


Figure 5: Data with removing seasonal trend

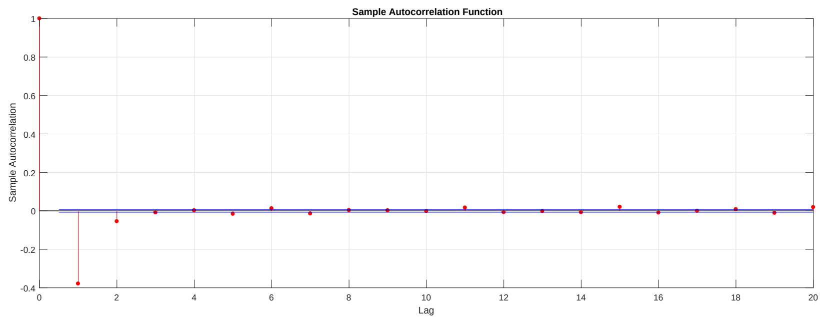
We use this error to find ARIMA model. This section conclude that the data has a periodic every 50 hours, 100 hours, 150 hours, 200 hour and 250 hours.

4.2 Differencing

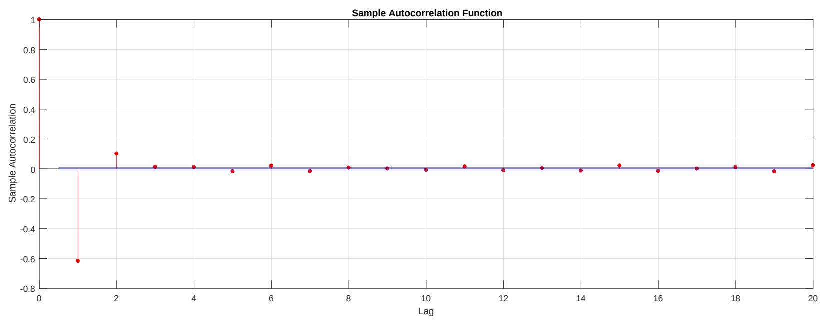
After removing the seasonal trend, the autocorrelation function(ACF) of the estimation error are shown in figure 6(a). Figure 6(a) show that there is non-stationary data because autocorrelation function is depend on time. So that we must differentiate the data before using maximum likelihood estimation to find the ARMA model. The autocorrelation function of one time differencing data and two time differencing data are shown in figure 6.
From figure 6, we can conclude that $d = 1$ is the best choice because it has more similar to white noise than $d = 2$.



(a) without differencing



(b) 1 time differencing



(c) 2 times differencing

Figure 6: ACF of both one and two time differencing

4.3 Estimation of the ARMA model and Model Selection

From the two section above, we choose $SARIMA(p, 1, q)(0, 1, 1)_{1000}$.
This section we use maximum likelihood estimation to estimate the parameter . Then use each model to find AIC and BIC scores for finding the optimal model.
The result of both AIC and BIC score and RMSE are shown in table 4.3.
From the table 4.3 show that $SARIMA(3, 1, 6)(0, 1, 1)_{1000}$ has the least AIC and BIC score. Thus, we choose $SARIMA(3, 1, 6)(0, 1, 1)_{1000}$ to forecast the data which can write into equation 31.

p,q	RMSE on validation data set	AIC($\times 10^5$)	BIC($\times 10^5$)
1,1	127.9871	8.4790	8.4791
1,2	127.8967	8.4789	8.4792
1,3	127.8874	8.4789	8.4793
1,4	128.0809	8.4769	8.4773
1,5	127.8158	8.4788	8.4794
1,6	127.8368	8.4788	8.4795
2,1	150.6522	9.0281	9.0284
2,2	127.9526	8.4790	8.4793
2,3	127.8996	8.4789	8.4794
2,4	127.8824	8.4789	8.4795
2,5	128.0494	8.4786	8.4793
2,6	127.6021	8.4755	8.4762
3,1	127.8743	8.4789	8.4793
3,2	127.8466	8.4762	8.4766
3,3	127.7314	8.4758	8.4764
3,4	127.9108	8.4787	8.4793
3,5	127.9746	8.4786	8.4794
3,6	127.4815	8.4714	8.4722
4,1	127.8806	8.4789	8.4794
4,2	127.8780	8.4788	8.4795
4,3	127.9541	8.4753	8.4792
4,4	128.0137	8.4786	8.4793
4,5	127.5999	8.4753	8.4761
4,6	127.9630	8.4787	8.4796

Table 1: Candidate score and RMSE of each order

$$\begin{aligned}(1-L^{1000})(1-L)(1-(1.29L+0.42L^2-0.71L^3))I(t) = \\ (1-0.91L^{1000}))(1-1.82L+0.18L^2+L^3-0.29L^4-0.03L^5-0.04L^6))e(t)\end{aligned} \tag{31}$$

5 Conclusions

The result in the experimant show that we can find the solar irradiance model by using time series model which the process is following in figure 3. This model is conclude a seasonal trend which has 50 hours seasonal period. Finally, the equation 31 can write the h-step forecasting equation which are shown in equation 32

$$\begin{aligned}\hat{I}(t+h|t) = &-0.29\hat{I}(t+h-1|t)+1.71\hat{I}(t+h-2|t)-0.29\hat{I}(t+h-3|t)-0.71\hat{I}(t+h-4|t)\\ &+0.29\hat{I}(t+h-1001|t)-1.71\hat{I}(t+h-1002|t)+0.29\hat{I}(t+h-1003|t)+0.71\hat{I}(t+h-1004|t)\\ &+e(t+h)-1.82e(t+h-1)+0.18e(t-2)+e(t+h-3)-0.29e(t+h-4)\\ &-0.03e(t+h-5)-0.04e(t+h-6)-0.91e(t+h-1000)+1.6562e(t+h-1001)\\ &-0.1638e(t+h-1002)-0.91e(t+h-1003)+0.2639e(t+h-1004)\\ &+0.0273e(t+h-1005)+0.0364e(t+h-1006)\end{aligned} \tag{32}$$

6 Appendix

6.1 Matlab Code

Find power spectrum density

```
% Find power spectral density
y=fft(IMean5m);
N=numel(y);

% Use FFT to find power spectrum density
PSD = abs(y).^2/N;
PSD = PSD/max(PSD);
logPSD = log(PSD);
f=1/N*(1:N);
plot(f(1:N/2),logPSD(1:N/2))
```

Find the seasonal trend

```
%Fitting Seasonal Trend
t=1:length(IMean5m);
t=t';
A=[cos(0.002*pi*t) cos(0.004*pi*t) cos(0.006*pi*t) cos(0.008*pi*t) cos(0.01*pi*t) sin(0.002*pi*t) sin(0.004*pi*t) sin(0.006*pi*t) sin(0.008*pi*t) sin(0.01*pi*t) ones(length(IMean5m),1)];
xls = A\IMean5m;
yest = A*xls;

resid = IMean5m-yest;
```

Find the order d

```
% create differencing operator
D1=LagOp({1 -1},'Lags',[0,1]);
% Differentiate 1 time
diff1=filter(D1,resid);
autocorr(diff1)
```

Find the candidate models

```
% Split to training data set and validation data set
dataTra = IMean(1:72400);
dataVad = IMean(length(dataTra)+1:length(IMean));

% p and q are the highest order of AR and MA polynomial
SpecMdl = arima('Constant',0,'ARLags',1:p,'D',1,'MALags',1:q,'Seasonality',1000,'SMALags',1000);

[EstMdl,EstParamCov,logL,info] = estimate(SpecMdl,dataTra);
% Find AIC and BIC
AIC(p,q) = -2*logL+2*(4+q);
BIC(p,q) = -2*logL+(4+q)*log(N);
% Find residual error
[Et,Vt] = infer(EstMdl,dataTra);
MAEt(p,q)=sum(abs(Et))/length(dataTra);
RMSEt(p,q)=sqrt(sum(Et.^2)/length(dataTra));
% Find estimated error
[Ev,Vv] = infer(EstMdl,dataVad);
```

```
MAEv(p,q)=sum(abs(Ev))/length(dataVad);
RMSEv(p,q)=sqrt(sum(Ev.^2)/length(dataVad));
```

References

[1] J. D. Hamilton. *Time Series Analysis.* Princeton University Press, 1994.

[2] W. Khovidhungij. *Signal, Systems and Control.* Chulalongkorn University Press, 2016.

[3] V. Layanun and J. Songsiri. Solar irradiance forecasting for chulalongkorn university location using time series models. http://jitkomut.eng.chula.ac.th/group/vichaya_report.pdf, 2016.

[4] L. Ljung. *System Identification:Theory for user.* Prentice-Hall, 1987.

[5] T. Söderström and P. Stoica. *System Identification.* Prentice Hall International, 1989.