2102531 System Identification

An Identification of Building Temperature System
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Abstract

In this project, we aim to estimate the system matrices of the real-world building temperature system via system identification. The building model we considered has 2 rooms which have heat interactions with walking path and ambient. Only one-dimensional heat transfer and internal energy change have been concerned while neglecting effects caused by humidity, solar irradiance, and air leakage. An air-conditioners system is described by first law of thermodynamics and coefficient of performance (COP) and is expressed as a continuous-time dynamic equation. Since we found that a state-space equation becomes a nonlinear function, then we propose two models in this project, which are linearlized model approximate linear model with assumptions that the COP is a constant obtained by the air-conditioners specifications. Since the data measured by sensors is discretized, then we define a discrete-time equivalent state- space model by using zero-order-hold equivalent. Temperature data and air-conditioners electrical input data can be collected by the Chulalongkorn University Building Energy Management System (CUBEMS) [1]. Since the temperature data and electrical input data can be measured, we choose a least-squares estimation with constraints as an estimation method. The results show that the dynamic matrix obtained via least-squares method is stable. Input matrix is forced to have a same structure as a state-space equation we have derived. In addition, we validate our model with a validation data set. The results show that the two models we estimated provide a fitting with moderate performance.

1 Background

1.1 Air-conditioning system

Air-conditioning system is a heat-removing system widely used in many applications. Basic principles of air-conditioning systems are (i) removing heat from the system to ambient (ii) manipulating an air flow and humidity (iii) keeping the temperature consistent at the setpoint temperature.

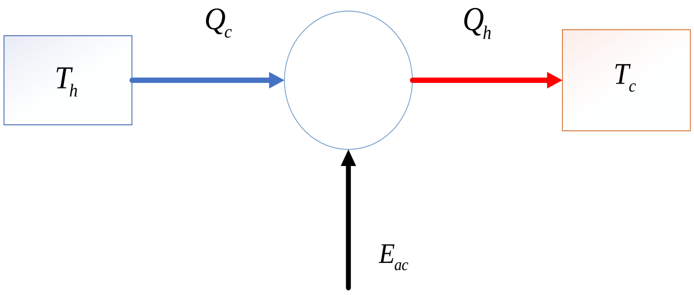


Figure 1: Diagram of air-conditioners system

The relationship between electrical input power and heat absorbed by air conditioner can be derived by using the first law of thermodynamics

$$Q_c + E_{ac} = Q_h, \tag{1}$$

where

- Q_c is cooling flow generated by air conditioner.
- Q_h is heat pulled out by air conditioner.
- E_{ac} is an electrical energy input.

1.2 Coefficient of performance

Define a coefficient of performance (COP) as a performance index of an air-conditioner [1] [2] [3], i.e,

$$\mathrm{COP} = \frac{Q_c}{E_{ac}}, \tag{2}$$

where

- Q_c is cooling flow generated by an air-conditioner (kWh).
- E_{ac} is an electrical energy used by an air-conditioner(kWh).

1.2.1 Maximum theoritical coefficient of performance

Assume that a thermodynamical process of air-conditioners is the reversed Carnot cycle.

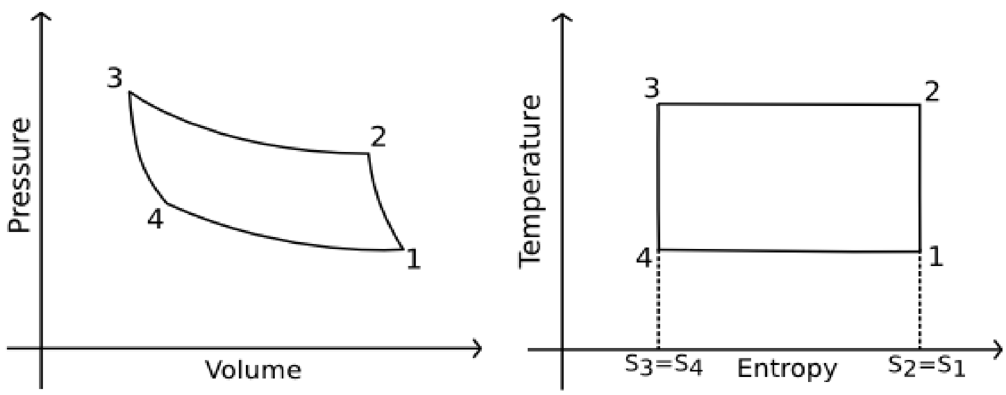


Figure 2: The reversed Carnot cycle
(from `http://static.mtdevans.com/`)

The reversed Carnot cycle is said to be the most efficient cooling cycle which means we assume that air-conditioners operate at its maximum theoritical performance. Under the assumption that air conditioners can be described by the reversed carnot cycle. According to [1], we can use the carnot equation $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c} = \frac{T_\infty}{T}$ and (2) can be rewritten to

$$\mathrm{COP} = \frac{T(t)}{T_\infty(t) - T(t)}, \tag{3}$$

where

- $T(t)$ is current room temperature ($^\circ\mathrm{C}$).
- $T_\infty(t)$ is ambient temperature ($^\circ\mathrm{C}$).

1.2.2 Practical approximated coefficient of performance

In practical implementation, the maximum theoritical coefficient of performance cannot be reached due to loss in a cooling process. The coefficient of performance can be obtained by using experiments based on air-conditioner specifications described in [4] [5]. In this project, we introduce seasonal energy efficiency ratio (SEER) and energy efficiency ratio (EER) obtained by

$$\mathrm{SEER} = \frac{\text{Seasonal cooling capacity } (\mathrm{BTU})}{\text{total input energy } (\mathrm{W{\cdot}h})},$$
$$\mathrm{EER} = \frac{\text{Cooling ability } (\mathrm{BTU/h})}{\text{Input power } (\mathrm{W})}.$$

According to [6], a relationship between both effeciency ratios and coefficient of performance can be expressed as

$$\begin{aligned} \mathrm{EER} &= 1.12 \cdot \mathrm{SEER} + 0.02 \cdot \mathrm{SEER}^2, \\ \mathrm{COP} &= 0.293 \cdot \mathrm{EER}. \end{aligned} \tag{4}$$

1.3 Steady heat transfer

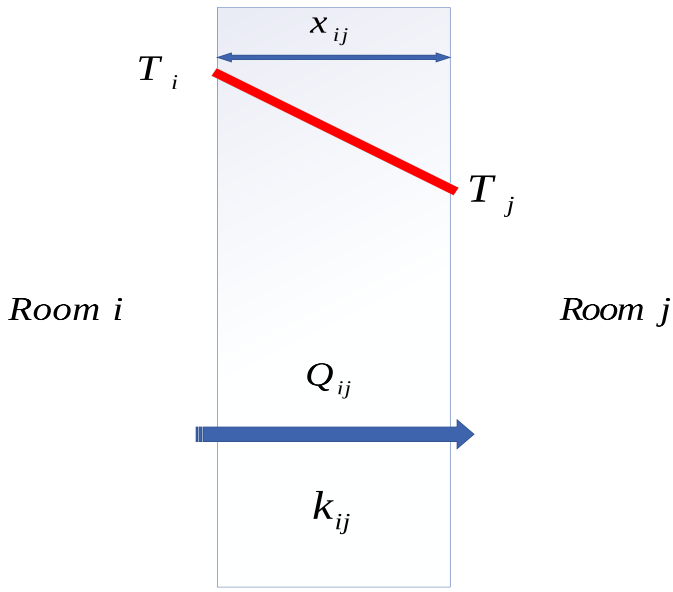


Figure 3: Steady heat transfer

We assume that there does not exist any heat sinks but the model only consists of one-dimensional heat conduction through a plane wall. According to [7] [8], the heat transfer can be described by using the one-dimensional Fourier's law as

$$\frac{d^2T}{dx^2} = 0.$$

Denote the temperature at two sides of a wall, $T_1(t)$ and $T_2(t)$. Therefore, we can derive the linear heat transfer equation as

$$\dot{Q}(t) = \frac{kA}{X}(T_1(t) - T_2(t)). \tag{5}$$

1.3.1 Inside-building effect

Inside the building, there exists two factors that cause heat flow: the temperature difference between two beside rooms and the temperature difference between each room and walkway. From (5), the heat transfer between two rooms can expressed as

$$\dot{Q}_{ij}(t) = \frac{k_{ij}A_{ij}(T_j(t) - T_i(t))}{X_{ij}} = \alpha_{ij}(T_j(t) - T_i(t)), \tag{6}$$

where

- Q_{ij} is heat transferred between room i^{th} and room j^{th}
- k_{ij} is a thermal conductivity of the beside wall.
- A_{ij} is an area of the wall.
- X_{ij} is a thickness of the wall.
- $T_i(t)$ is a room i^{th} current temperature ($^\circ$C).
- $T_j(t)$ is a room j^{th} current temperature ($^\circ$C).
- α_{ij} is a physical property constant of the wall.

And we can also derive the heat transfer from walkway to room i^{th} as

$$\dot{Q}_{wi}(t) = \frac{k_{wi}A_{wi}(T_w(t) - T_i(t))}{X_{wi}} = \alpha_{wi}(T_w(t) - T_i(t)), \tag{7}$$

where

- Q_{wi} is heat transferred from walkway hall to room i^{th}
- k is a thermal conductivity of the beside wall.
- A is an area of the beside wall.
- X_{wi} is a thickness of the wall.
- $T_i(t)$ is a room i^{th} current temperature ($^\circ$C).
- $T_w(t)$ is a walkway current temperature ($^\circ$C).
- α_{wi} is a physical property constant of the walkway-side wall.

1.3.2 Outside-building effect

In this study, we will consider only the heat transfer caused by the temperature diffence (ignore the solar irradiance effect). Therefore, we can also use (5) to write the heat transfer equation from ambient to room i^{th} as

$$\dot{Q}_{\infty i}(t) = \frac{k_{\infty i} A_{\infty i}(T_\infty(t) - T_i(t))}{X_{\infty i}} = \alpha_{\infty i}(T_\infty(t) - T_i(t)), \tag{8}$$

where

- $Q_{\infty i}$ is heat exchanged between room i^{th} and outside ambient.
- $k_{\infty i}$ is a thermal conductivity of the ambient-side wall.
- $A_{\infty i}$ is an area of the ambient-side wall.
- $X_{\infty i}$ is a thickness of the ambient-side wall.
- $\alpha_{\infty i}$ is a physical property constant of the ambient-side wall.
- $T_\infty(t)$ is an ambient current temperature ($^\circ\mathrm{C}$).

1.4 Sensible heat effect

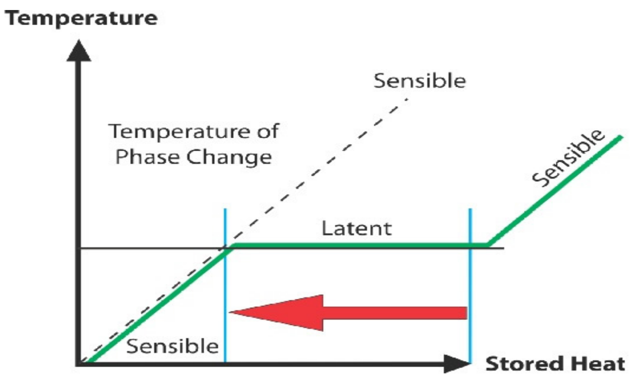


Figure 4: Phase change plot of the substance
(from `http://www.rgees.com/images/LHvsSH.jpg`)

Since there is no phase change in the building temperature system, the change of internal energy of closed system can be described as

$$\Delta U_i = \mathrm{m}_i c \Delta T_i, \tag{9}$$

where

- ΔU_i is the change of internal energy in room i^{th}.
- m_i is an air mass in room i^{th}.
- c is a specific heat constant of air.
- ΔT_i is the change in temperature in room i^{th} ($^\circ\mathrm{C}$).

2 System modelling

2.1 Building temperature system

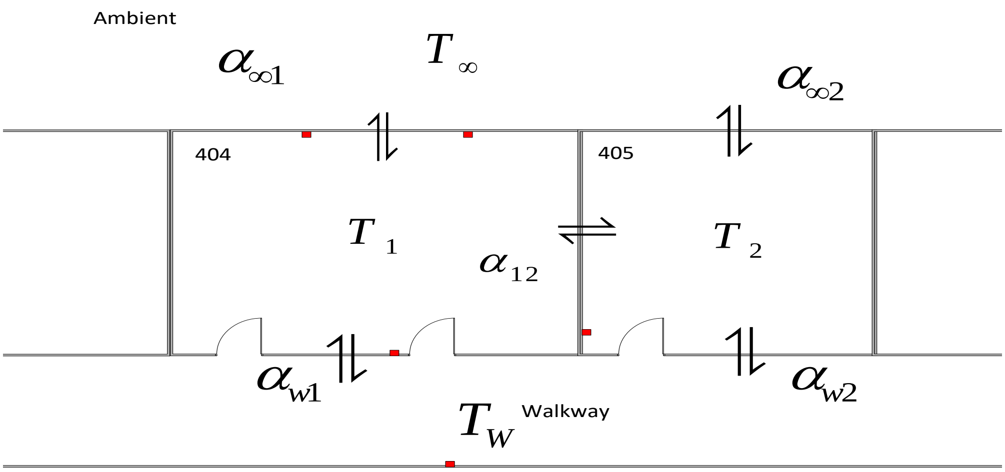


Figure 5: Building temperature system

The building that we considered is EE Building, floor 4, Department of engineering, Chulalongkorn university shown in figure 5.

2.2 Dynamic equation of the building temperature system

The first law of thermodynamics can be used to describe the building temperature system. Assume that the system is closed. We can use (1) and (9) to show that

$$\Delta U_i = m_i c \Delta T_i = Q_i + Q_{c,i}, \tag{10}$$

where

- ΔU_i is a change in an internal energy inside the room i^{th}.
- $Q_i = Q_{ij} + Q_{wi} + Q_{\infty i}$ is a total heat flow inside the room i^{th}.
- $Q_{c,i}$ is a total cooling flow generated by i^{th} air conditioner.

Apply the first derivative on both sides, we then have

$$m_i c \frac{d\Delta T_i}{dt} = \frac{dQ_i}{dt} + \frac{dQ_{c,i}}{dt}.$$

Because $\frac{d\Delta T_i}{dt} = \frac{d}{dt}(T_i - T_{set}) = \dot{T}_i$, we can write the dynamic equation as

$$m_i c \dot{T}_i = \dot{Q}_{ij} + \dot{Q}_{wi} + \dot{Q}_{\infty i} + \frac{d}{dt}(\mathrm{COP}_i E_{ac,i}), \tag{11}$$

where $\frac{d}{dt}(\mathrm{COP}_i E_{ac,i})$ can be expressed by using (2), i.e,

$$\begin{aligned} \frac{d}{dt}(\mathrm{COP}_i E_{ac,i}(t)) &= \frac{d}{dt}(\mathrm{COP}_i) E_{ac,i}(t) + \mathrm{COP}_i P_{ac,i}(t) \\ &= \frac{d}{dt}\left(\frac{T_i(t)}{T_\infty(t) - T_i(t)}\right) E_{ac,i}(t) + \frac{T_i(t)}{T_\infty(t) - T_i(t)} P_{ac,i}(t) \end{aligned} \tag{12}$$

2.3 State-space model of the building temperature system

2.3.1 Linearized model

Define state variables $x_1(t) = T_1(t) - \mathrm{T_{set}}$ and $x_2(t) = T_2(t) - \mathrm{T_{set}}$.From (11) and (12), It is obvious that COP_1 and COP_2 is a nonlinear function in state variables $T_1(t)$ and $T_2(t)$, thus we do a linearization.

Consider

$$\begin{aligned}
\dot{T}_1(t) &= \frac{(T_\infty(t) - T_1(t))^2}{m_1 c(T_\infty(t) - T_1(t))^2 - E_{ac,1}(t)}([-(\alpha_{12} + \alpha_{w1} + \alpha_{\infty 1})T_1(t) + \alpha_{12}T_2(t) + \alpha_{w1}T_w(t) + \alpha_{\infty 1}T_\infty(t)] \\
&\quad + \dot{T}_\infty(t)E_{ac,1}(t) + \frac{T_1(t)}{T_\infty(t) - T_1(t)}\dot{E}_{ac,1}(t)) \\
\dot{T}_2(t) &= \frac{(T_\infty(t) - T_2(t))^2}{m_2 c(T_\infty(t) - T_2(t))^2 - E_{ac,2}(t)}([-(\alpha_{12} + \alpha_{w1} + \alpha_{\infty 1})T_2(t) + \alpha_{12}T_1(t) + \alpha_{w1}T_w(t) + \alpha_{\infty 1}T_\infty(t)] \\
&\quad + \dot{T}_\infty(t)E_{ac,2}(t) + \frac{T_1(t)}{T_\infty(t) - T_1(t)}\dot{E}_{ac,2}(t))
\end{aligned} \tag{13}$$

Let $x = \begin{bmatrix} T_1(t) - \mathrm{T_{set}} & T_2(t) - \mathrm{T_{set}} \end{bmatrix}^T, u = \begin{bmatrix} E_{ac,1}(t) & E_{ac,2}(t) \end{bmatrix}^T, \dot{u} = \begin{bmatrix} P_{ac,1}(t) & P_{ac,2}(t) \end{bmatrix}^T$, and $w = \begin{bmatrix} T_\infty(t) & T_w(t) \end{bmatrix}.^T$ By using concepts of a Taylor expansion about point $(x_e, u_e, \dot{u}_e, w_e)$, let $\delta\dot{x} = \delta f(x, u, \dot{u}, w)$, Then we have

$$\begin{aligned}
\delta\dot{x} &= \underbrace{\frac{\partial}{\partial x} f(x, u, \dot{u}, w) \mid_{x_e, u_e, \dot{u}_e, w_e}}_{A}(x - x_e) + \underbrace{\frac{\partial}{\partial u} f(x, u, \dot{u}, w) \mid_{x_e, u_e, \dot{u}_e, w_e}}_{B}(u - u_e) \\
&\quad + \underbrace{\frac{\partial}{\partial \dot{u}} f(x, u, \dot{u}, w) \mid_{x_e, u_e, \dot{u}_e, w_e}}_{\bar{B}}(\dot{u} - \dot{u}_e) + \underbrace{\frac{\partial}{\partial w} f(x, u, \dot{u}, w) \mid_{x_e, u_e, \dot{u}_e, w_e}}_{\Gamma}(w - w_e)
\end{aligned} \tag{14}$$

After linearization, the result shows that system matrices structure become

$$\begin{aligned}
A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \qquad B_1 = \begin{bmatrix} b^1_{11} & 0 \\ 0 & b^1_{22} \end{bmatrix}, \qquad B_2 = \begin{bmatrix} b^2_{11} & 0 \\ 0 & b^2_{22} \end{bmatrix}, \\
C &= I, \qquad D = 0, \\
\Gamma &= \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}.
\end{aligned} \tag{15}$$

Thus, (15) can be rewritten in a standard notation as

$$\begin{aligned}
\dot{x} &= Ax + B_1 u + B_2 \dot{u} + \Gamma w \\
y &= x
\end{aligned} \tag{16}$$

where

- A is an dynamic matrix.
- B_1 and B_2 are an input matrix of u and $\dot{u}$.
- $C = I$ is an output matrix.
- $x = \begin{bmatrix} T_1(t) - T_{set} \\ T_2(t) - T_{set} \end{bmatrix}$ is a state vector, $u = \begin{bmatrix} E_{ac,1}(t) \\ E_{ac,2}(t) \end{bmatrix}$ and $\dot{u} = \begin{bmatrix} P_{ac,1}(t) \\ P_{ac,2}(t) \end{bmatrix}$are input vectors.
- $w = \begin{bmatrix} T_\infty(t) \\ T_w(t) \end{bmatrix}$ is a disturbance with a gain matrix Γ.

2.3.2 Approximate linear model

Assume that COP can be treated as a constant. Thus, nonlinearity of COP is negligible. To derive an approximate state-space equation, define $x_1 = T_1(t) - \mathrm{T_{set}}$, and $x_2 = T_2(t) - \mathrm{T_{set}}$ where $T_1(t)$, $T_2(t)$, and $\mathrm{T_{set}}$ are room 404 current temperature, room 405 current temperature, and setpoint temperature respectively. According to [9], we can use (6), (7), (8), and (11) to derive a state-space equation of the building temperature system in a matrix form as

$$\begin{bmatrix} \dot{T}_1(t) \\ \dot{T}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{(\alpha_{12}+\alpha_{w1}+\alpha_{\infty1})}{m_1 c} & -\frac{\alpha_{12}}{m_1 c} \\ -\frac{\alpha_{12}}{m_2 c} & -\frac{(\alpha_{12}+\alpha_{w2}+\alpha_{\infty2})}{m_2 c} \end{bmatrix} \begin{bmatrix} T_1(t) - \mathrm{T_{set}} \\ T_2(t) - \mathrm{T_{set}} \end{bmatrix} + \begin{bmatrix} \frac{\mathrm{COP}_1}{m_1 c} & 0 \\ 0 & \frac{\mathrm{COP}_2}{m_2 c} \end{bmatrix} \begin{bmatrix} P_{ac,1} \\ P_{ac,2} \end{bmatrix} + \begin{bmatrix} \frac{\alpha_{\infty1}}{m_1 c} & \frac{\alpha_{w1}}{m_1 c} \\ \frac{\alpha_{\infty2}}{m_2 c} & \frac{\alpha_{w2}}{m_2 c} \end{bmatrix} \begin{bmatrix} T_\infty(t) \\ T_w(t) \end{bmatrix}. \tag{17}$$

Thus, (17) can be rewritten in a standard notation as

$$\begin{aligned} \dot{x} &= Ax + Bu + \Gamma w, \\ y &= x, \end{aligned} \tag{18}$$

where

- A is an dynamic matrix.
- B is an input matrix of u.
- $C = I$ is an output matrix.
- $x = \begin{bmatrix} T_1(t) - \mathrm{T_{set}} \\ T_2(t) - \mathrm{T_{set}} \end{bmatrix}$ is a state vector, $u = \begin{bmatrix} P_{ac,1}(t) \\ P_{ac,2}(t) \end{bmatrix}$ is input vectors.
- $w = \begin{bmatrix} T_\infty(t) \\ T_w(t) \end{bmatrix}$ is a disturbance with a disturbance matrix Γ.

2.4 Discrete state-space model of building temperature system

Since the data measured by sensors is discretized with the sampling rate h, we have defined to a discrete-time equivalent state-space model. In this project, we introduce (i) zero-order-hold equivalent (ii) forward-Euler method.
Let

$$H(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

and h be a sampling time.

- Zero-order-hold equivalent is computed by

$$H(z) = \begin{bmatrix} e^{Ah} & \int_0^h e^{A\tau} B d\tau \\ C & D \end{bmatrix},$$

  or

$$H(z) = (1 - z^{-1}) \mathcal{Z}\{\mathcal{L}^{-1}\{\frac{H(s)}{s}\}\}.$$

- Forward-Euler method is computed by subtituting

$$s = \frac{z-1}{h}.$$

After applied ZOH equivalent and forward-Euler method on (16) and (18), we found that the structure of both system of the system matrices can be described as

- Linearized model

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B_1 = \begin{bmatrix} b_{11,1} & 0 \\ 0 & b_{22,1} \end{bmatrix}, B_2 = \begin{bmatrix} b_{11,2} & 0 \\ 0 & b_{22,2} \end{bmatrix}, \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix},$$

- Approximate linear model

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}, \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix},$$

with

$$C = I, D = 0.$$

As results, the stuctures of discrete-time system matrices of these two model are same as (16) and (18).

3 Data

3.1 Variables and parameters

Table 1: Variables and parameters of the building temperature system

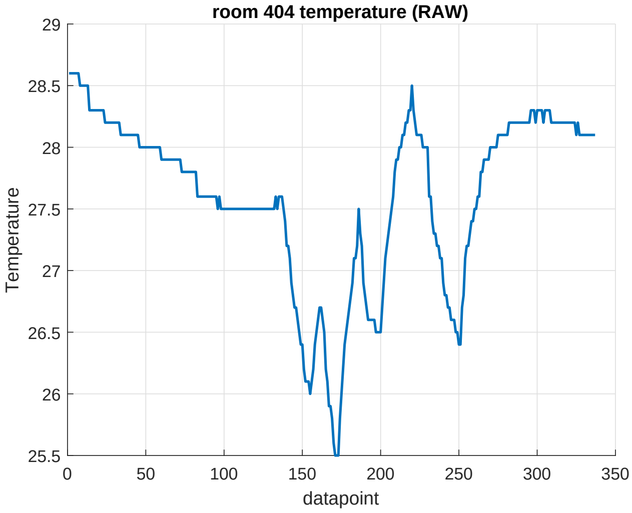
variables		parameters	
measured	unknown	measured	unknown
$T_1(t), T_2(t)$ $T_\infty(t)$ $T_w(t)$ $E_{ac,1}(t), E_{ac,2}(t)$	$P_{ac,1}(t), P_{ac,2}(t)$	$\mathrm{T}_{\mathrm{set}}$ c $\mathrm{COP}_1, \mathrm{COP}_2$	α_{12} $\alpha_{\infty 1}, \alpha_{\infty 2}$ α_{w1}, α_{w2} m_i c

Table 1 shows variables and parameters of the building temperature system and classifies those into 2 classes: measurable class and unknown class.

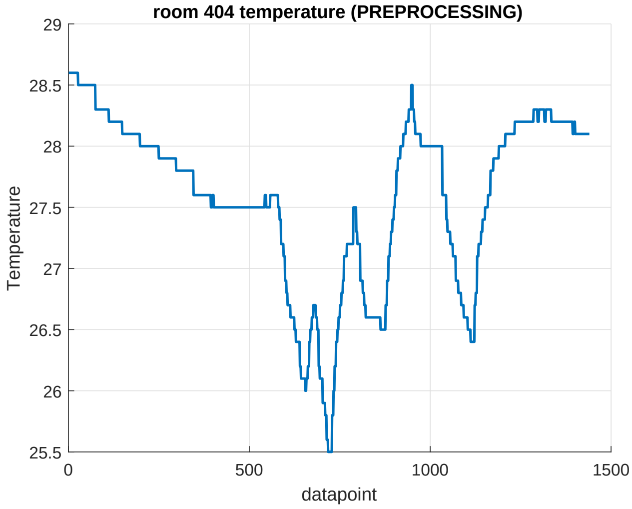
3.2 Preprocessing

Since we collected temperature of room 404 and 405, ambient temperature, walk-way temperature, and input energy of air-conditioner from [10] while almost all data have one-minute sampling time. Thus, we will adjust sampling rate corresponding to all measured data at 1 minute.

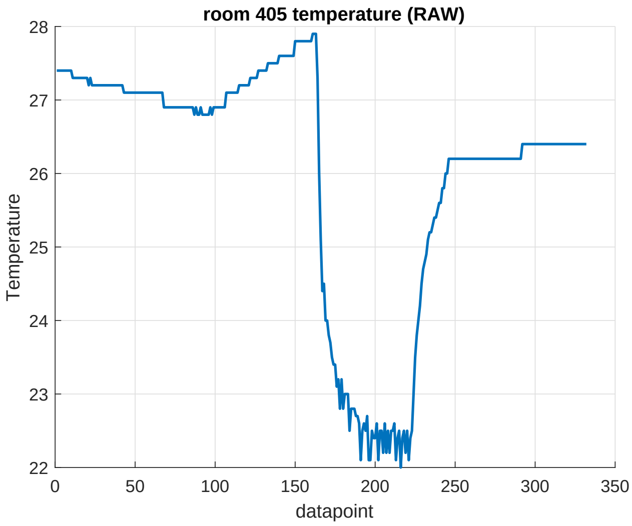
Since room 404 temperature and 405 temperature do not have a constant rate of sampling and their sampling rates are not equal to 1. Thus we will assume that some data that missing are equal to the past data as follows.



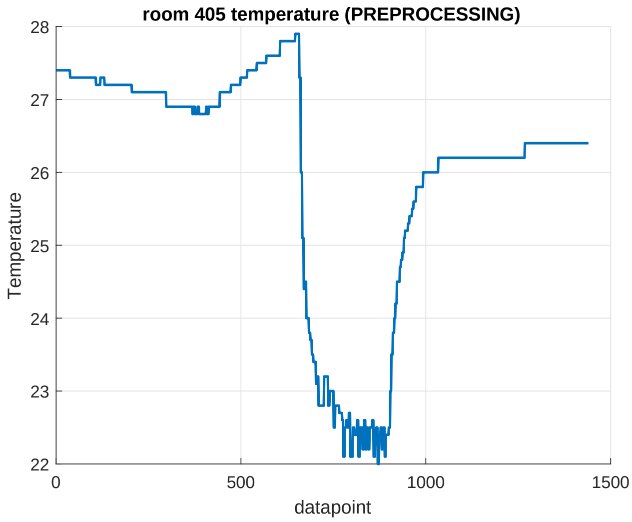
(a) Raw data of room 404 temperature



(b) 1 minute sampling time of room 404 temperature



(c) Raw data of room 405 temperature

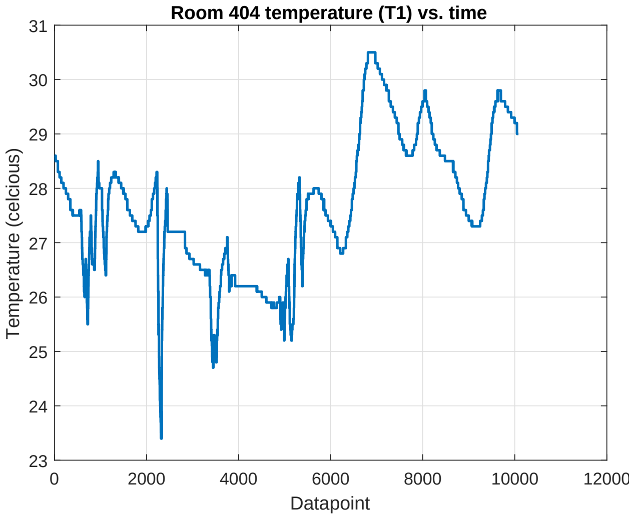


(d) 1 minute sampling time of room 405 temperature

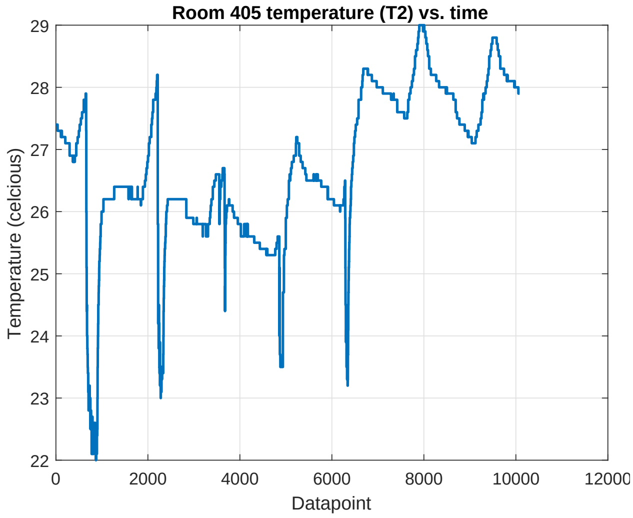
3.3 Data

Since we adjust settling time at 1 minute, each one-week data set contains 10,080 points. All temperature data, and electricity input data are collected during 16 Oct 2017 - 22 Oct 2017 (one-week data) from [10] shown as follows.

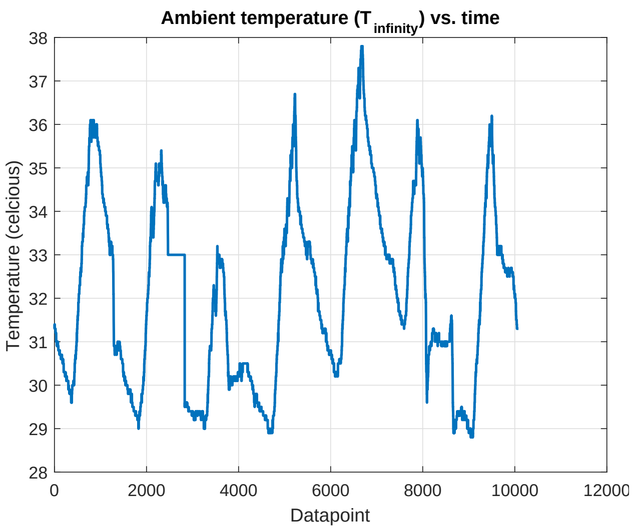
- Temperature data



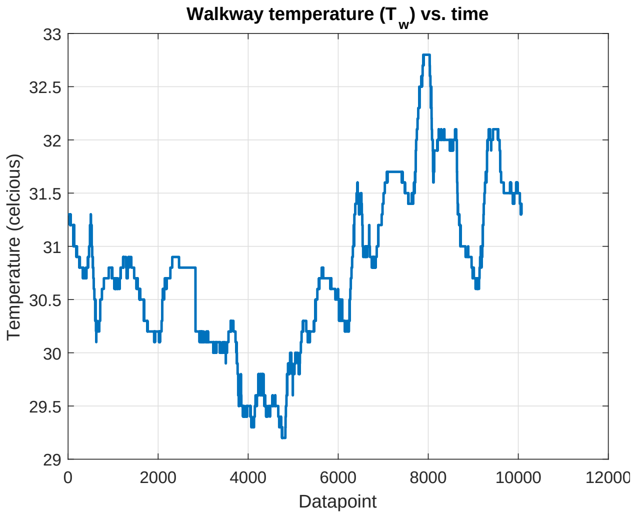
(e) Room 404 temperature



(f) Room 405 temperature

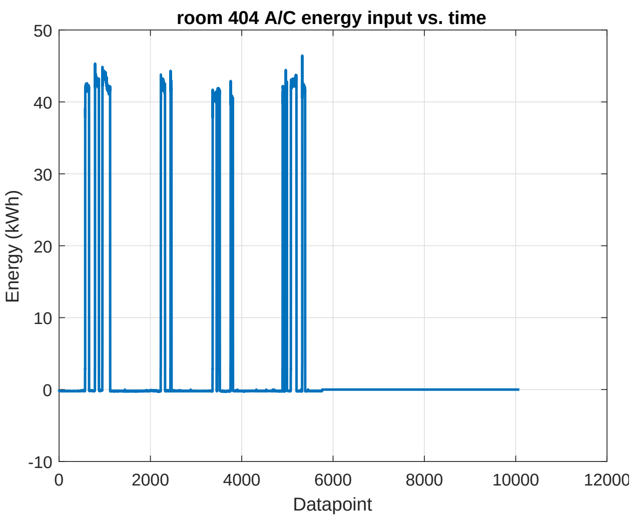


(g) Ambient temperature

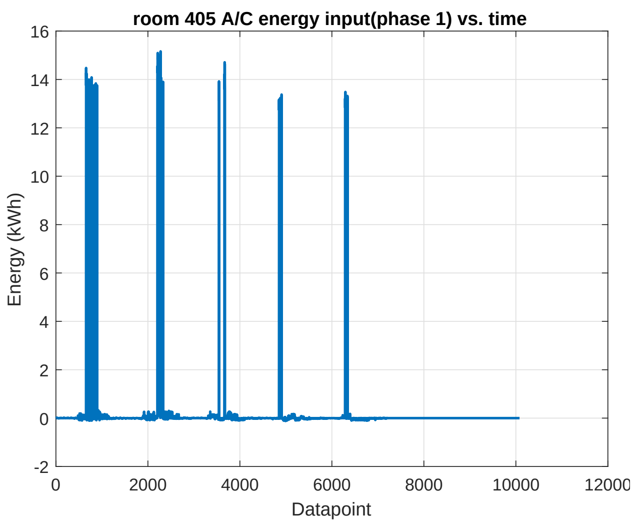


(h) Walkway temperature

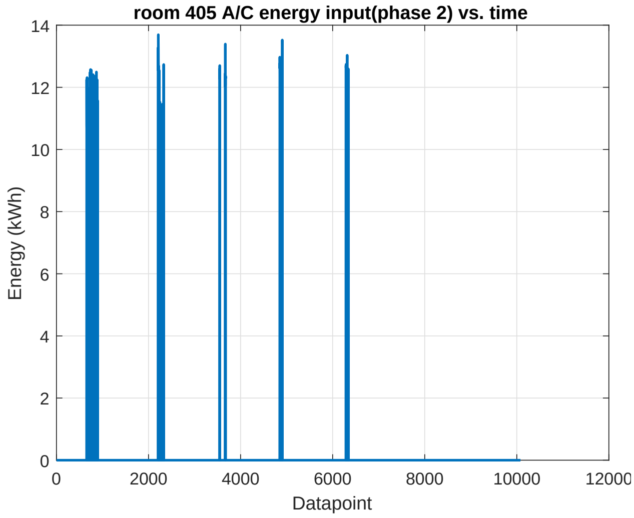
- Energy input data



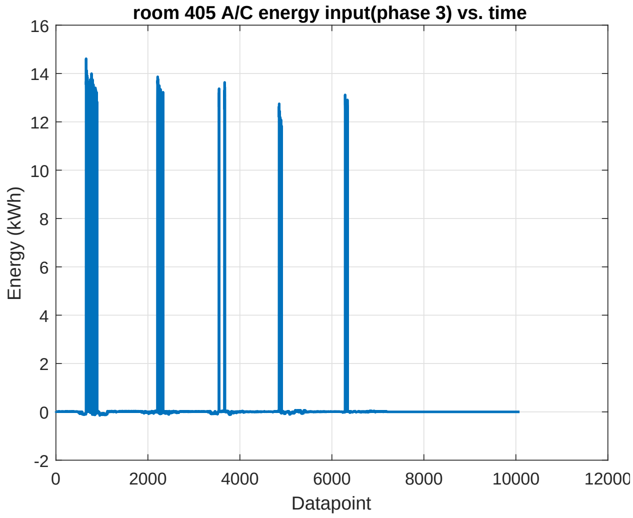
(i) Room 404 A/C energy input



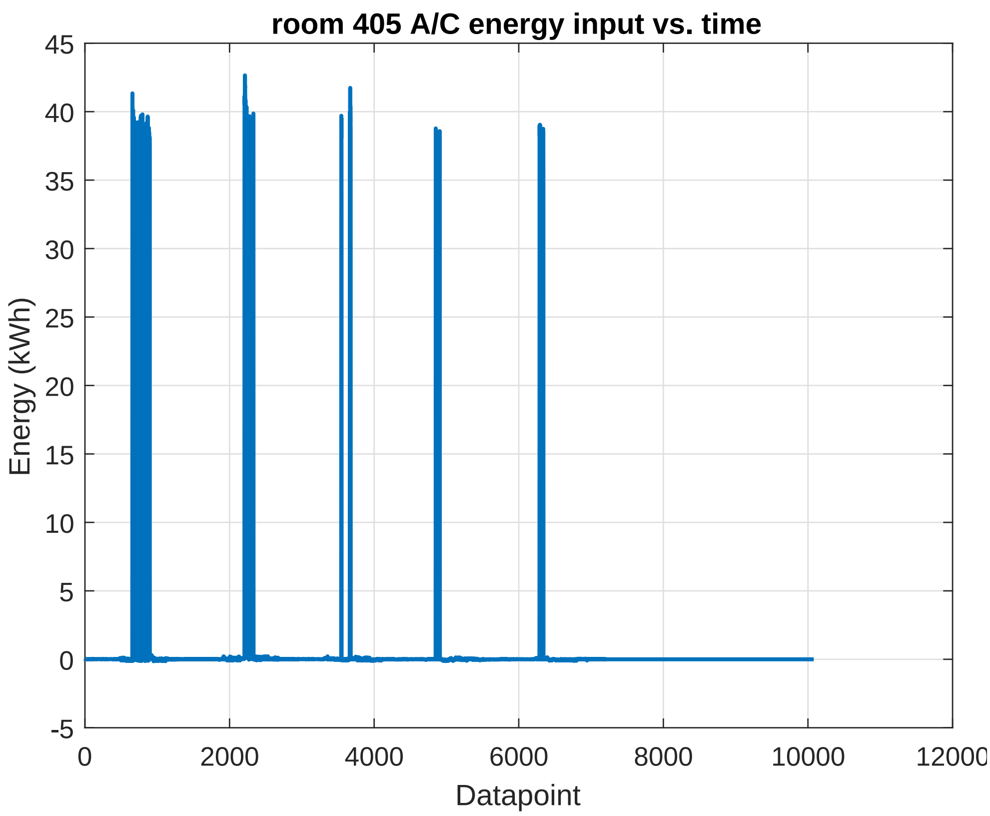
(j) Room 405 A/C energy input (phase 1)



(k) Room 404 A/C energy input (phase 2)



(l) Room 404 A/C energy input (phase 3)



(m) Room 404 A/C energy input (total)

4 Problem statement

In this project, we use system identification to determine the discrete-time system matrices in section 2.4, then transform them back to continuous-time system. The models we aim to identify are (16), and (18). Since $T_1[t]$ and $T_2[t]$ which are both states and outputs can be measured. Therefore, we can use a least-squares estimation with constraints to determine the discrete-time system matrices.

4.1 Least-squares estimation

In this project, room temperature, ambient temperature, walkway temperature, electrical input energy which also leads to approximate electrical input power can be measured directly via sensors, Thus we can simply use these measurements to formulate least-squares problems to determine discrete-time system matrices as follows.

Least-squares problem formulation

- linearized model in section 2.3.1

$$\begin{aligned} \min_{\hat{A},\hat{B}_1,\hat{B}_2,\hat{\Gamma}} & \left\| x[t+1] - \hat{A}x[t] - \hat{B}_1 u[t] - \hat{B}_2 \dot{u}[t] - \hat{\Gamma} w[t] \right\|^2 \\ \text{s.t.} \quad & \text{Structures of } \hat{B}_1, \hat{B}_2, \hat{C}, \hat{D} \text{ are fixed.} \end{aligned} \tag{19}$$

- approximate linear model in section 2.3.2

$$\begin{aligned} \min_{\hat{A},\hat{B},\hat{\Gamma}} & \left\| x[t+1] - \hat{A}x[t] - \hat{B}u[t] - \hat{\Gamma} w[t] \right\|^2 \\ \text{s.t.} \quad & \text{Structures of } \hat{B}, \hat{C}, \hat{D} \text{ are fixed.} \end{aligned} \tag{20}$$

5 Experiments

First, we use the one-week data (10,800 samples) in section 3.3 as a model-training set. Assume that the state sequences are generated by (16) and (18), then we can apply a least-squares method to determine the system matrices under the constraints. In these experiments, we used 2 sets of data, a training set, and a validation set collected during 23-29 Oct 2017. After solving least-squares problems with constraints by using `CVX` toolbox, state sequences had been generated via system matrices we have obtained.

5.1 Result

5.1.1 Discrete-time system matrices

the discrete-time system matrices obtained from `CVX` are shown as follows. Optimal values were calculated by $\|Qx - p\|^2$ shown in table 2 where Q is $\begin{bmatrix}\text{room temperature data} & \text{electrical input data} & \text{disturbance data}\end{bmatrix}$, p is a state matrix, and x is an augmented system matrices.

Table 2: values of the objective function at optimal points

Data set	Linearized model	Approximate linear model
Optimal value	11.9677	11.9677

- Linearized model

$$\begin{aligned} \hat{A} &= \begin{bmatrix} 0.9929 & 0.0005 \\ 0.0019 & 0.9895 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} -0.0009 & 0 \\ 0 & -0.0033 \end{bmatrix} \\ \hat{B}_2 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \hat{\Gamma} = \begin{bmatrix} 0.0034 & 0.0015 \\ 0.0011 & 0.0071 \end{bmatrix}. \end{aligned} \tag{21}$$

  Eigenvalues of $\hat{A} = 0.9932, 0.9892$. ($\hat{A}$ is stable.)

- Approximated linear model

$$\hat{A} = \begin{bmatrix} 0.9929 & 0.0005 \\ 0.0019 & 0.9895 \end{bmatrix}, \hat{B} = \begin{bmatrix} -0.0592 & 0 \\ 0 & -0.2036 \end{bmatrix}, \hat{\Gamma} = \begin{bmatrix} 0.0034 & 0.0015 \\ 0.0011 & 0.0071 \end{bmatrix}. \tag{22}$$

  Eigenvalues of $\hat{A} = 0.9932, 0.9892$. ($\hat{A}$ is stable.)

5.2 Fittings based on linearized model

In this section, the fitting based on linearized model on a validation data set has been shown as follows.

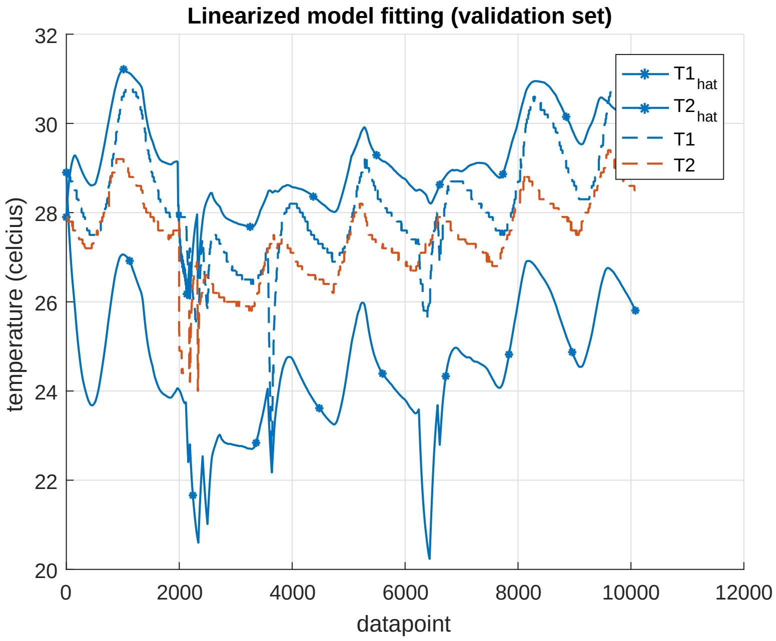


Figure 6: room temperature plot of a linearized model on a validation set

5.3 Fittings based on approximate linear model

In this section, the fitting based on approximate linear model on a validation data set has been shown as follows.

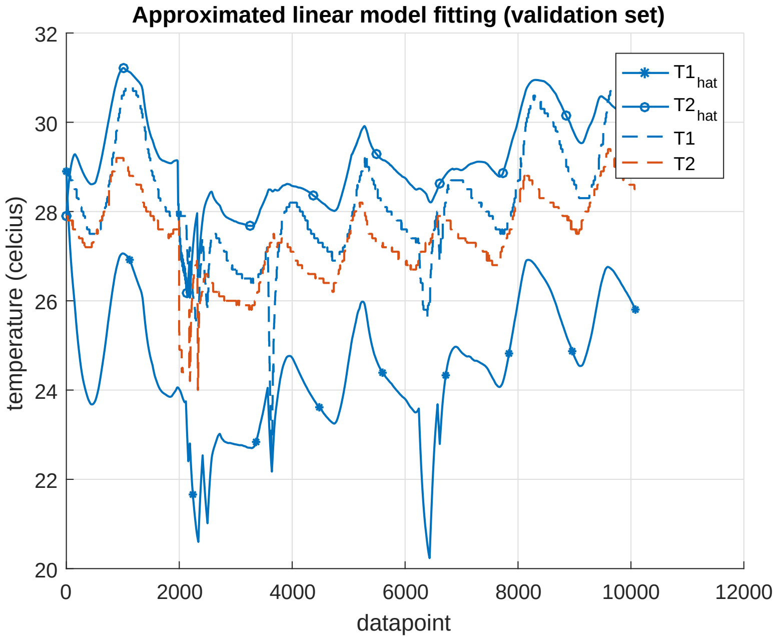


Figure 7: room temperature plot of an approximated linear model on a validation set

5.4 Discussion

After solving least-squares estimation with constraints, we found that the dynamic matrix we had obtained is stable ($\|\lambda\| < 1$, where λ is an eigenvalue of the dynamic matrix). Since the input matrix B_2 of the linearized model equals to a zero matrix, it shows that electrical energy input and electrical power input are dependent which are obviously shown in the section 3.2. The results show that both linearized model and approximate linear model provide moderate fitting performance.

6 Conclusions

In this project, we aim to estimate system matrices of the building temperature system by using system identification. The data we measured are temperature data, and electrical input data. After we formulate a state-space equation and determine a structure of system matrices, system matrices can be obtained by a least-squares estimation. In this project, we propose that the building temperature system can be described by two different models which are linearized model and approximate linear model. The results show that approximate linear model provides a better fitting performance than the linearized model since it gives a smaller value of mean-squares errors.

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7 Appendices

7.1 Appendix A: MATLAB code

- Least-squares estimation with constraints

```
clc; clear all;
%==========Training data set==========
datat = load('trainingdata.mat');
%number of data of training set
lt = length(datat.room404Temp);
%Temperature data of EE404 of training set
T1t(1:lt,1) = datat.room404Temp(1:lt);
%Temperature data of EE405 of training set
T2t(1:lt,1) = datat.room405Temp(1:lt);
%Temperature data of ambient of training set
Tinft =  datat.ambientTemp(1:lt);
%Temperature data of walkway of training set
Twt = datat.W(1:lt);

%==========Validation data set==========
datav = load('validdata.mat');
%number of data of validation set
lv = length(datav.vr404);
%Temperature data of EE404 of validation set
T1v = datav.vr404(1:lv);
%Temperature data of EE405 of validation set
T2v = datav.vr405(1:lv);
%Temperature data of ambient of validation set
Tinfv = datav.vambient(1:lv);
%Temperature data of walway of validation set
Twv = datav.vw(1:lv);

%=====load power========
P1t(1:lt,1) = datat.room404Energy(1:lt).*(1/60);
P2t(1:lt,1) = (datat.EE1 + datat.EE2 + datat.EE3).*(1/60);
P1v(1:lv,1) = datav.vE404(1:lv).*(1/60);
P2v(1:lv,1) = datav.vsumee.*(1/60);

%======load energy======
E1t(1:lt,1) = datat.room404Energy;
E2t(1:lt,1) = datat.EE1 + datat.EE2 + datat.EE3;
E1v(1:lv,1) = datav.vE404(1:lt);
E2v(1:lv,1) = datav.vsumee;

%==========linear==============
Q = [T1t(1:lt-1,1) T2t(1:lt-1,1) E1t(1:lt-1,1) E2t(1:lt-1,1) ...
     P1t(1:lt-1,1) P2t(1:lt-1,1) Tinft(1:lt-1,1) Twt(1:lt-1,1)];
p = [T1t(2:lt,1) T2t(2:lt,1)];
%cvx_begin
%     variable x(8,2)
%     minimize( norm( Q * x - p ) )
%     subject to
%     x(3,2) == 0
```

```
%     x(4,1) == 0
%     x(5,2) == 0
%     x(6,1) == 0
%cvx_end
% x(:,1) = lsqlin(Q,p(:,1),[0 0 0 0 0 0 0 0],0,[0 0 0 1 0 0 0 0;0 0 0 0 0 1 0 0],[0;0])
% x(:,2) = lsqlin(Q,p(:,2),[0 0 0 0 0 0 0 0],0,[0 0 1 0 0 0 0 0;0 0 0 0 1 0 0 0],[0;0])
% Al = x(1:2,:); Bl_1=x(3:4,:); Bl_2 =x(5:6,:); gammal = x(7:8,:);

%==========Approximate==============
R = [T1t(1:lt-1,1) T2t(1:lt-1,1) P1t(1:lt-1,1) P2t(1:lt-1,1) ...
       Tinft(1:lt-1,1) Twt(1:lt-1,1)];
p = [T1t(2:lt,1) T2t(2:lt,1)];
% cvx_begin
%      variable x(6,2)
%      minimize( norm( R * x - p ) )
%      subject to
%      x(3,2) == 0
%      x(4,1) == 0
% cvx_end
% y(:,1) = lsqlin(R,p(:,1),[0 0 0 0 0 0 ],0,[0 0 0 1 0 0 ],0)
% y(:,2) = lsqlin(R,p(:,2),[0 0 0 0 0 0 ],0,[0 0 1 0 0 0 ],0)
% Aa = x(1:2,:); Ba=x(3:4,:); gammaa = x(5:6,:);
```

- Preprocessing for EE404 room

```
%Sampling data to 1 miute of data in room404

%==========Initial parameter===========
day = 7; % use data of 1 week
EE404 = zeros(24*60*day,1); %empty vector of energy of room404
RT404 = zeros(24*60*day,1); %empty vector of room temperature 404
AB = zeros(24*60*day,1); %empty vector of ambient temperature
WW = zeros(24*60*day,1);%empty vector of walkway temperature
DT = datetime(DateTime);%DateTime is column vector recieve from CUBEMS
l = 1;
[a b] = size(DT);
[q r] = size(date); %date is colume vector from generatetime

%============Filtrate data==================
for i = 1:a
    D = day(DT(i));
    H = hour(DT(i));
    MT = minute(DT(i));
    for j = l:q
        D1 = day(date(j));
        H1 = hour(date(j));
        MT1 = minute(date(j));
        if H == H1 && MT == MT1 &&D == D1 %the data is that time we need
       EE404(j) = energy(i); %colume vector of energy recieve from CUBEMS
```

```
        RT404(j) = room(i);%colume vector of room temperature of room404 recieve from CUBEMS
        AB(j) = ambient(i);%colume vector of ambient temperature recieve from CUBEMS
        WW(j) = walkway(i)%colume vector of walkway recieve from CUBEMS
        l = j+1;
         break
         end
     end
end
[x y] = size(EE404);
for i = 2:x
     if EE404(i)== 0
         EE404(i) = EE404(i-1);
     end
end
for i = 2:x
     if AB(i)== 0
         AB(i) = AB(i-1);
     end
end
for i = 2:x
     if WW(i)== 0
         WW(i) =WW(i-1);
     end
end
for i = 2:x
     if RT404(i)== 0
         RT404(i) = RT404(i-1);
     end
end
```