

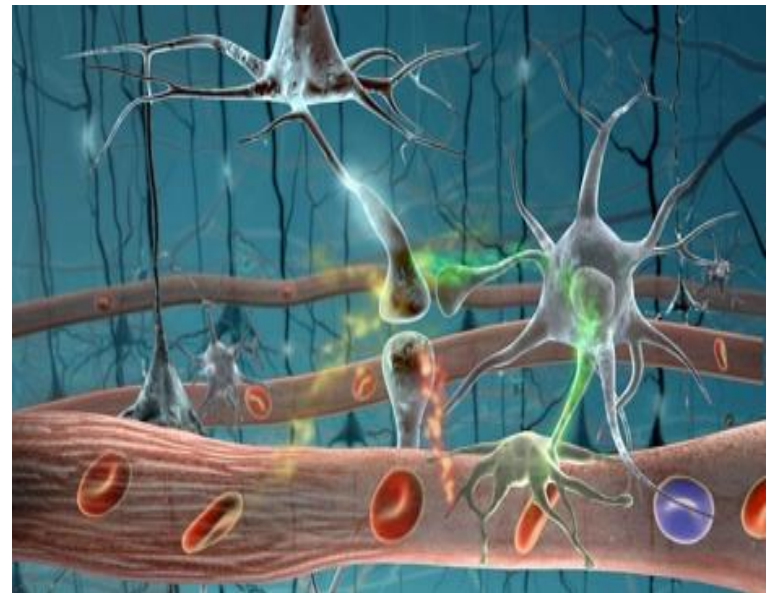
# Learning Causal Structures of Brain Connectivity in fMRI time series

Mr. Arnan Pongrattanakul 5230619021

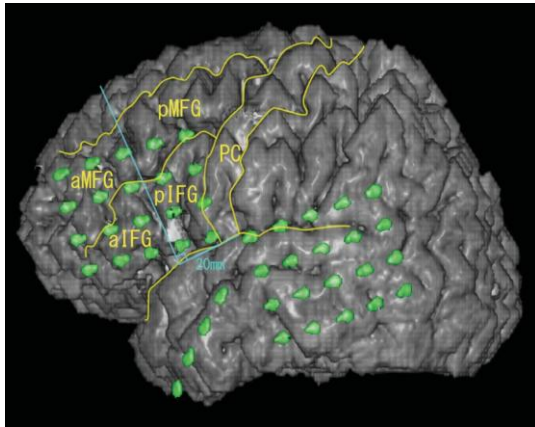
fMRI Scanner



BOLD



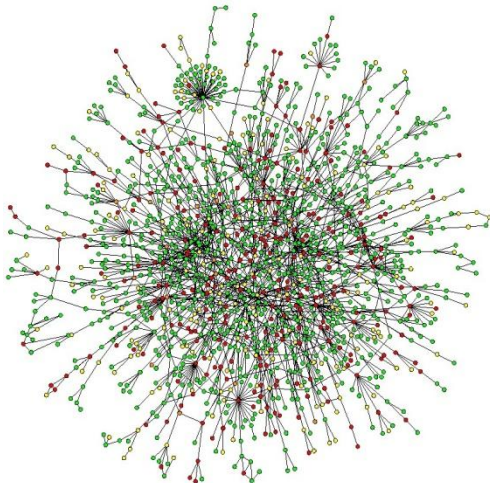
# fMRI



อ้างอิงจาก [www.neurosurgery-blog.com](http://www.neurosurgery-blog.com)



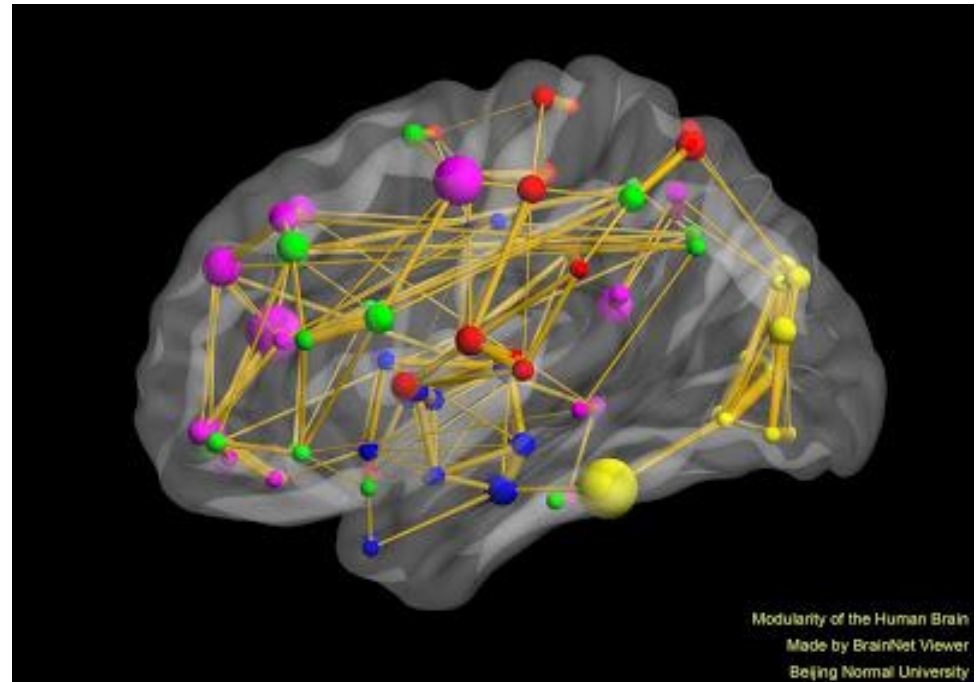
**Graphical model**



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# Objective

**Goal**



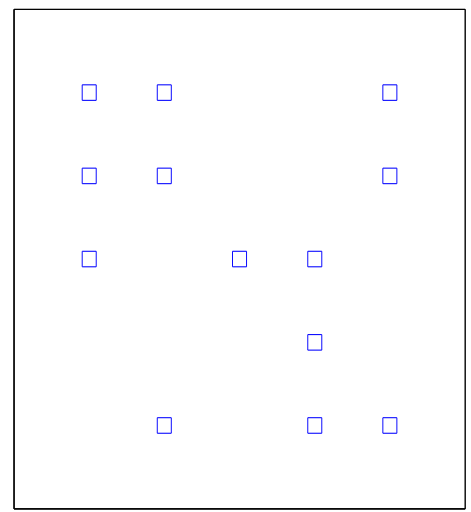
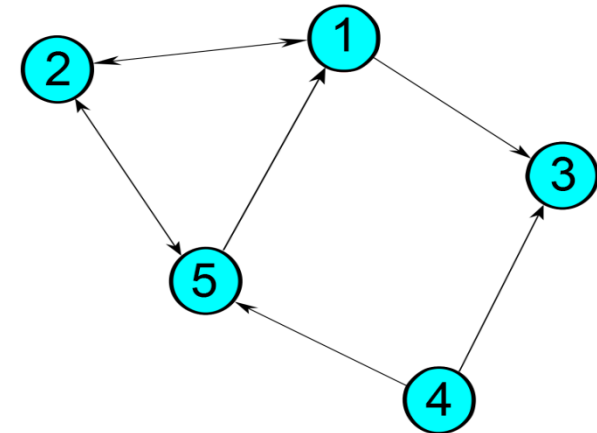
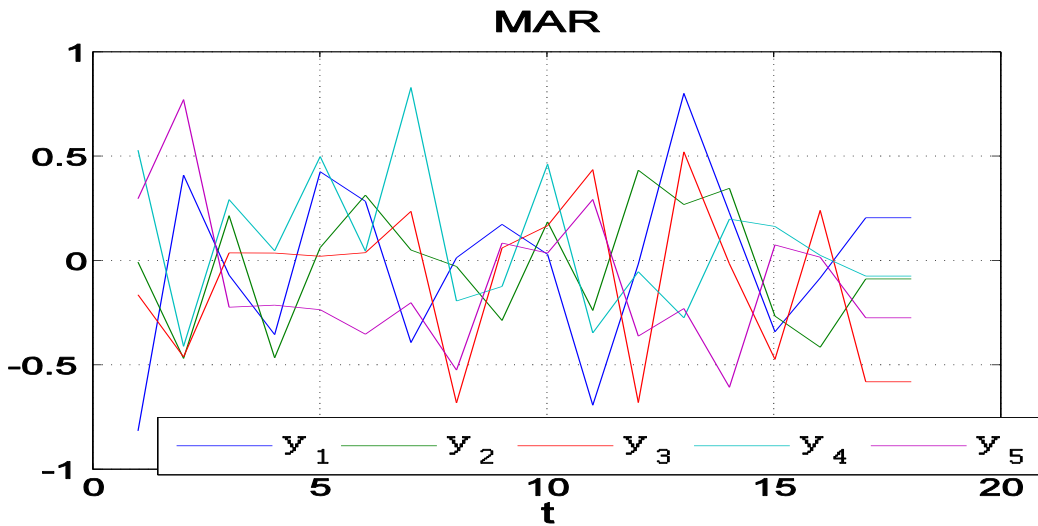
Modularity of the Human Brain  
Made by BrainNet Viewer  
Beijing Normal University

อ้างอิงจาก [psychophysiology.blogspot.com](http://psychophysiology.blogspot.com)

# MAR Model with Granger causality

$$y(t) = A_1 y(t-1) + A_2 y(t-2) + \dots + A_p y(t-p) + u(t)$$

$$y \in \mathbb{R}^n, A_k \in \mathbb{R}^{n \times n}, k = 1, 2, \dots, p$$



ถ้า  $y_j$  ไม่ใช่สาเหตุแบบ Granger ต่อ  $y_i$  แล้ว

$$[A_k]_{ij} = 0$$

# Optimization

Least-squares

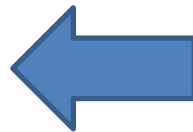
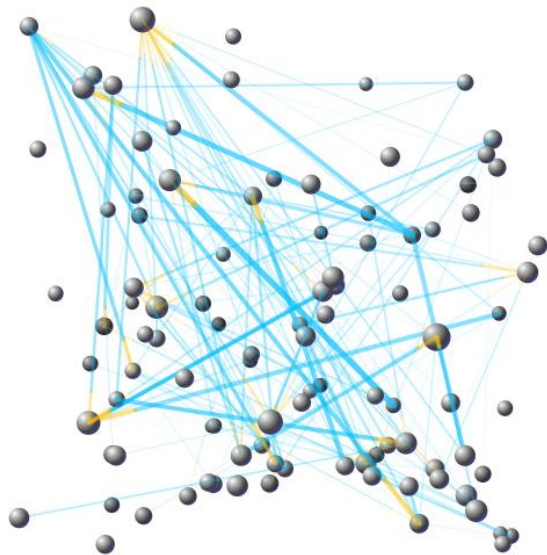
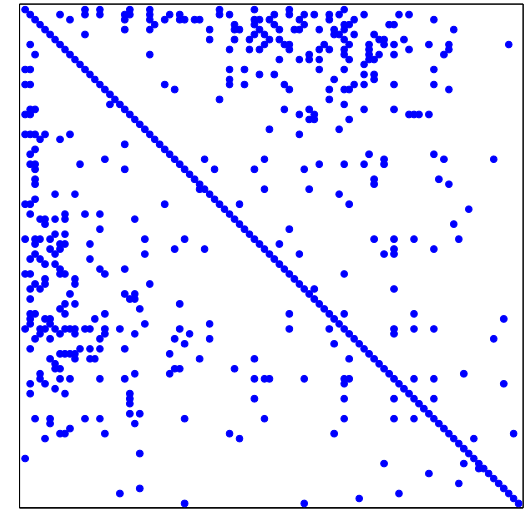
minimize  $(1/2) \|Y - AH\|_F^2$



Regularized least-squares

minimize  $(1/2) \|Y - AH\|_F^2 + \gamma g(A)$

$$g(A) = \sum_{i \neq j} \left\| \begin{bmatrix} (A_1)_{ij} & (A_2)_{ij} & \cdots & (A_p)_{ij} \end{bmatrix} \right\|_2$$



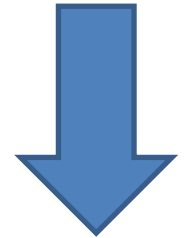
Least-squares with constrain

minimize  $(1/2) \|Y - AH\|_F^2$

$$(1/2) \|Y - AH\|_F^2$$

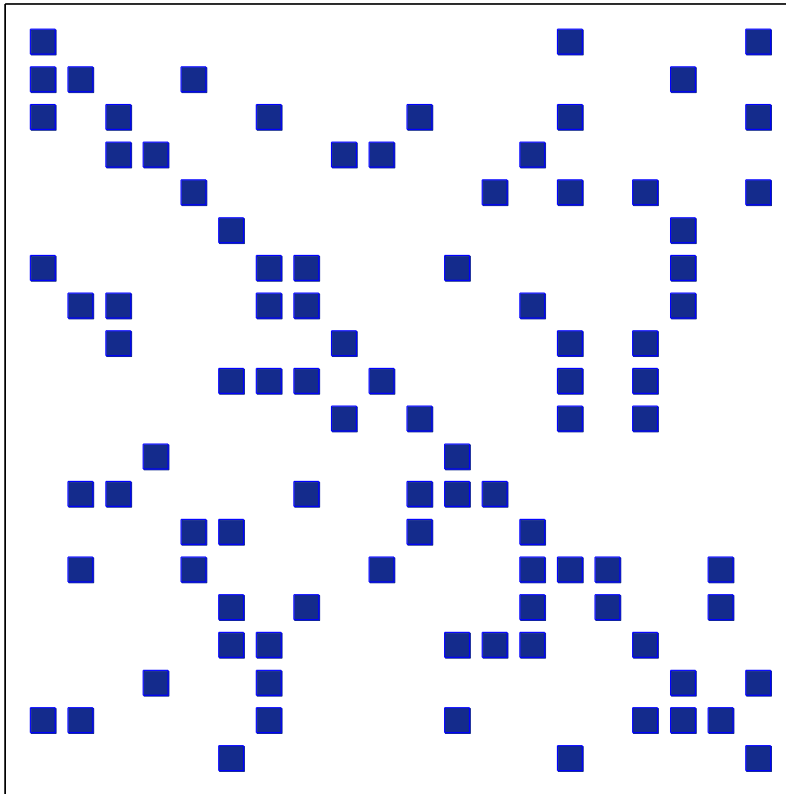
subject to

$$(A_1)_{ij} = \cdots = (A_p)_{ij} = 0, (i, j) \notin \mathcal{V}$$

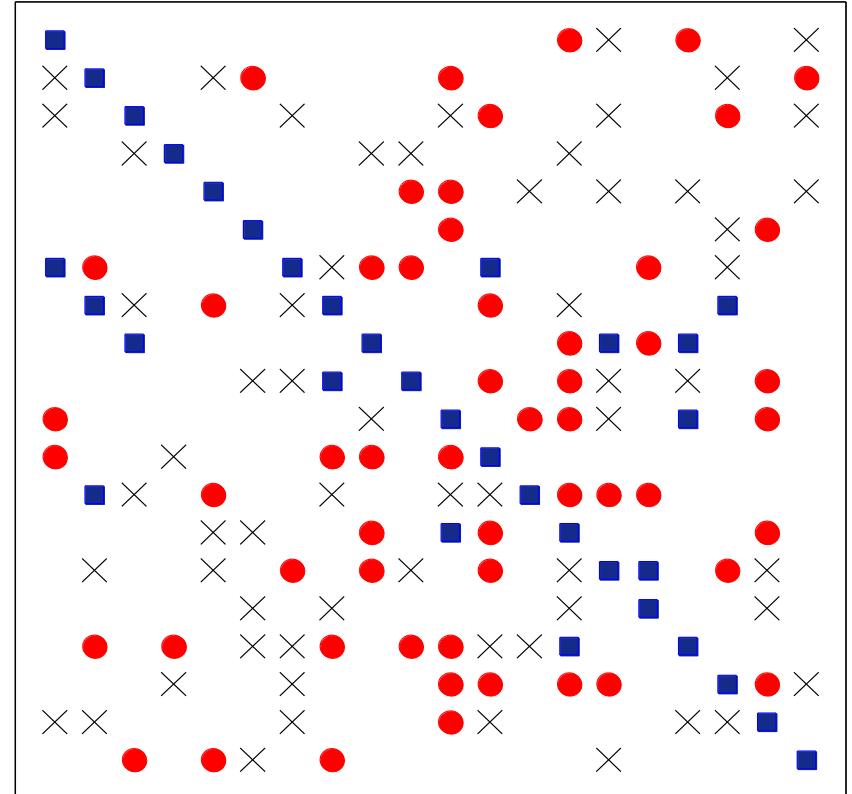


# Topology error

True



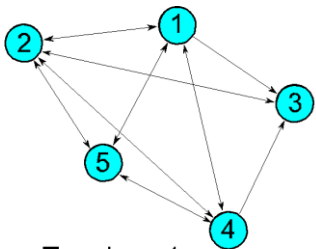
Estimate  
error = 30.5263%



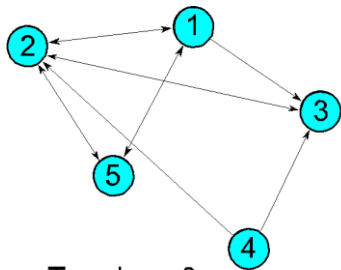
#FN = 61 #FP = 55

$$\text{error} = \frac{\# \text{mismatch zeros} + \# \text{mismatch nonzeros}}{n^2 - n}$$

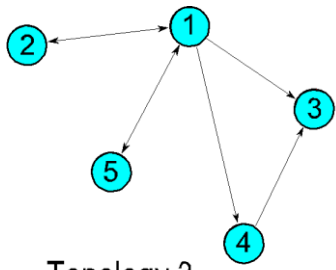
# Topology Selection



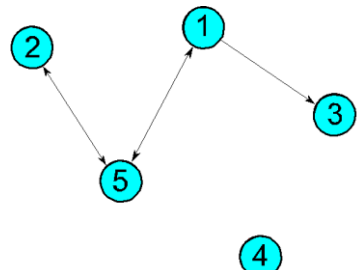
Topology 1



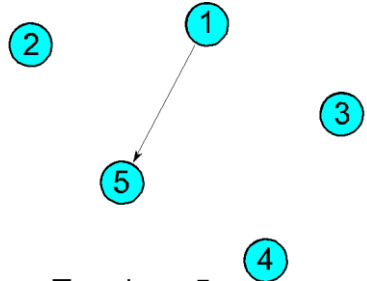
Topology 2



Topology 3



Topology 4

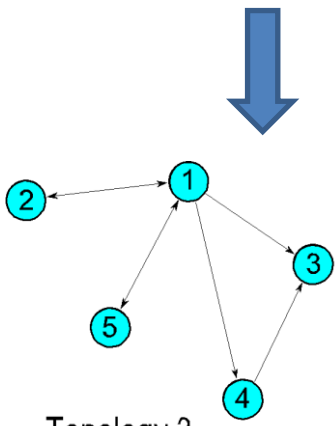


Topology 5

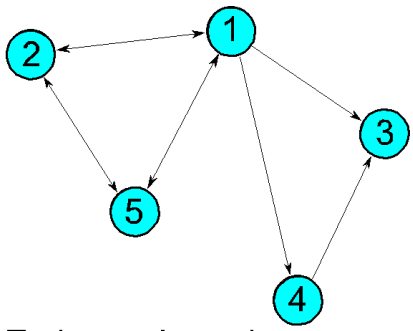
BIC score =  $-2\mathcal{L} + d \log(N)$   
 $\mathcal{L} = -(N/2) \log \det(R(\theta))$   
 $R(\theta) = (1/N)(Y - \hat{A}H)(Y - \hat{A}H)^T$



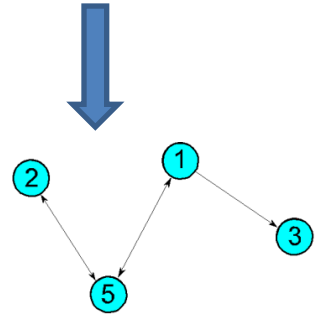
relative error =  $\frac{\|Y - \hat{A}H\|_F}{\|Y\|_F} \times 100\%$



Topology 3

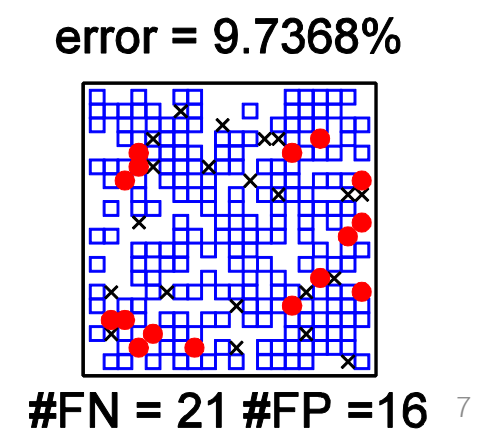
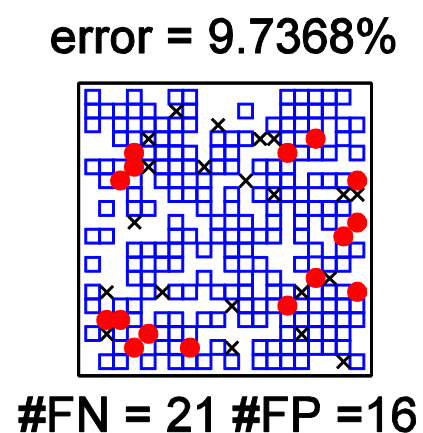
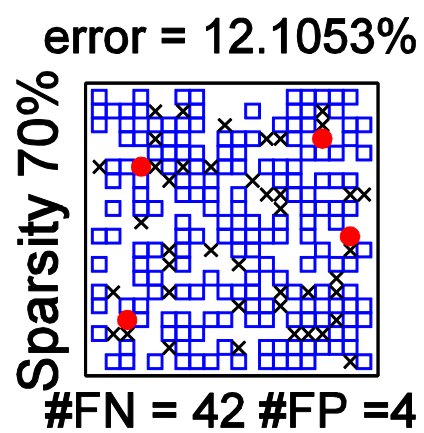
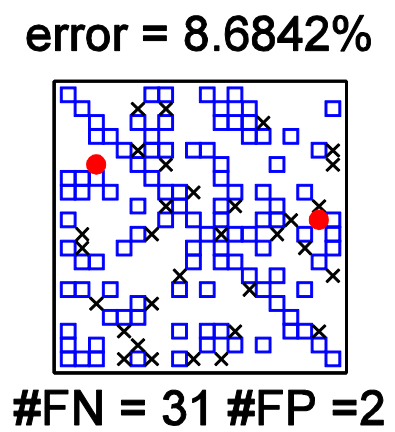
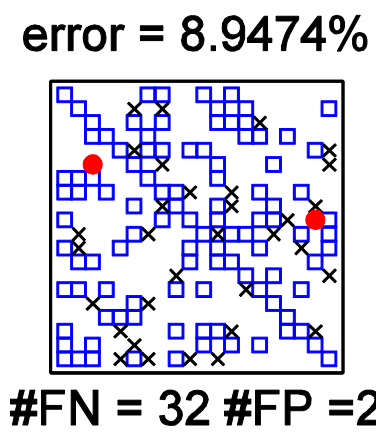
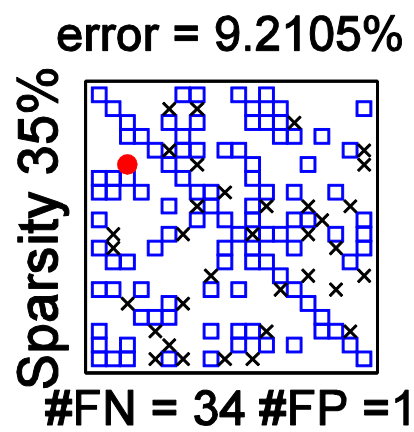
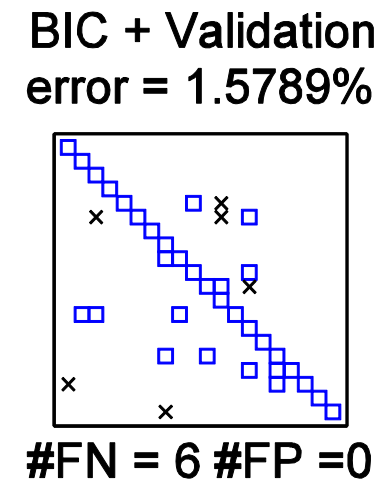
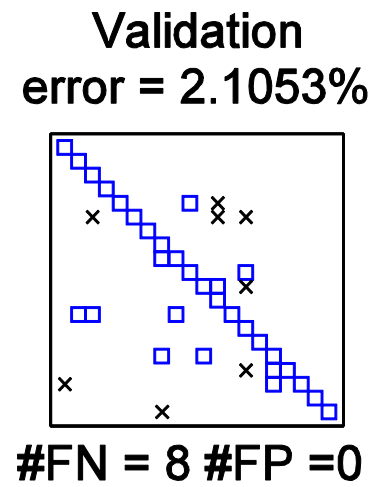
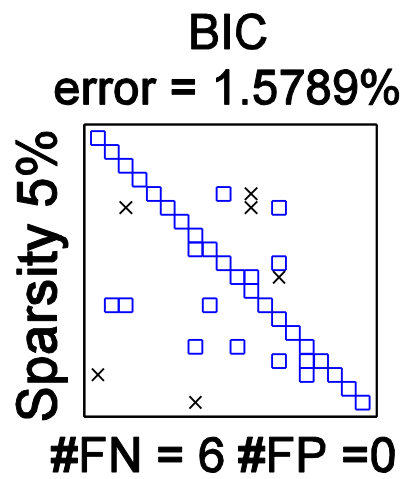


Estimated topology

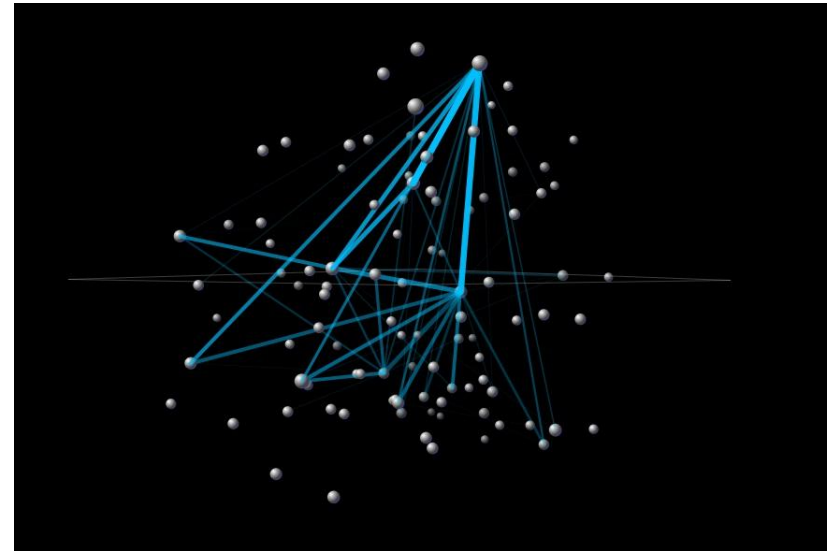
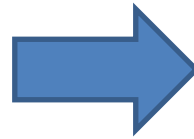
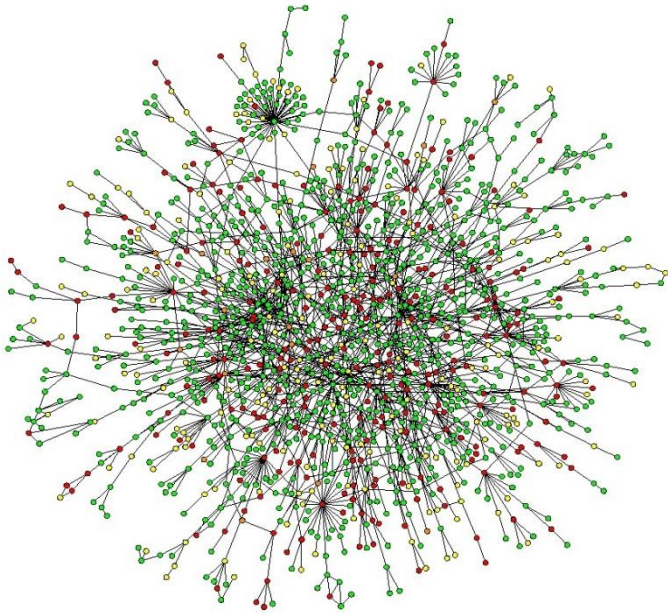


Topology 4

**E  
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# Dimension Reduction

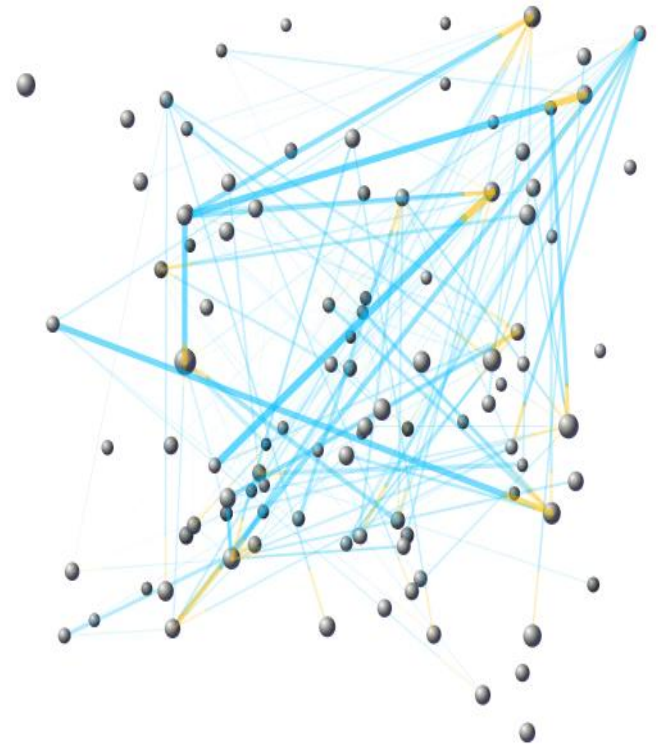
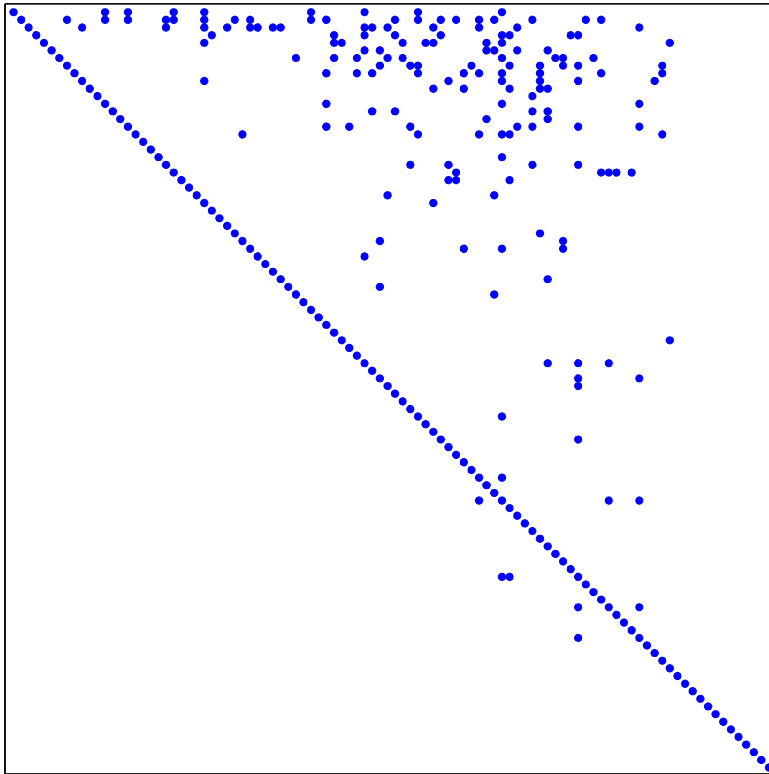


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PCA equation  $y = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \end{bmatrix}^T (x - \bar{x})$



# Graphical Model of fMRI



Q&A