Granger causality analysis of task-related fMRI time series

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Overview

1. functional magnetic resonance imaging
   - How to get fMRI data
   - Relationship between fMRI data and brain activity
2. Dynamic model of brain activity
   - Granger causal model
   - Including input term into Granger causal model
   - Least-square estimation
3. Preliminary Results
4. Project overview

Objective

- to study how to apply linear model to explain brain activity
- to develop numerical algorithm to solve model estimation problem

functional magnetic resonance imaging

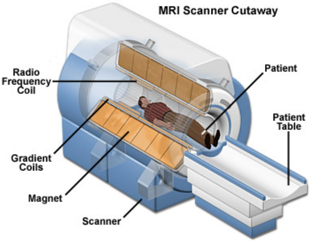


Figure: MRI scanner
Ref :
`http://www.clipmass.com/story/35886.`

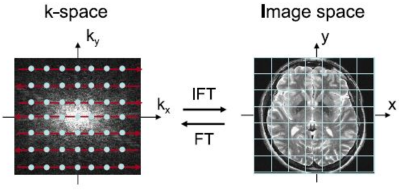


Figure: How to get MRI
Ref : M.A.Lindquist, J.M.Loh, L.Y.Atlas, and T.D.Wager, "Modeling the hemodynamic response function in fMRI : efficiency, bias and mis-modeling,"

Blood-Oxygen-Level-Dependent signal

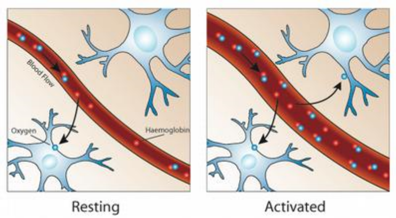


Figure: BOLD signal
Ref :
`www.sbirc.ed.ac.uk/research/techniques/functional.html`

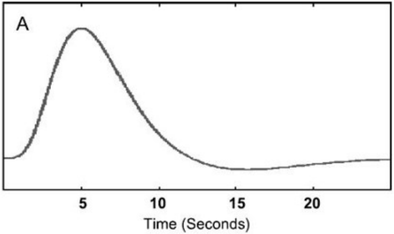


Figure: observed BOLD signal in voxel
Ref : M.A.Lindquist, J.M.Loh, L.Y.Atlas, and T.D.Wager, "Modeling the hemodynamic response function in fMRI: efficiency, bias and mis-modeling,"

Granger causal model

Granger causal model is one of models which use to explain brain activity. It use Granger causality and autoregressive model (AR) [A.Pruttiakaravanich, 2016]

$$y(t) = A_1 y(t-1) + A_2 y(t-2) + \ldots + A_p y(t-p) + e(t) \qquad (1)$$

where $y(\cdot) \in \mathbf{R}^n$ is the observed BOLD signal, $A_k \in \mathbf{R}^{n \times n}$, $k = 1, 2, \ldots, p$ and $e(\cdot)$ is noise.The concept of Granger causality is that if $y_j(t)$ is Granger-caused of $y_i(t)$, Information of previous $y_j(t)$ help to predict $y_i(t)$. In autoregressive process, the concept of Granger causality become simple condition that $y_j(t)$ isn't Granger causal $y_i(t)$ if and only if

$$(A_k)_{ij} = 0 \quad k = 1, 2, \ldots, p \qquad (2)$$

Granger causal model with Stimulus Input

Autoregressive model (1) does not have any input term but exogenous inputs could help to improve the prediction. We add input terms to the autoregressive model that changes to be autoregressive with Exogenous input model (ARX).

$$y(t) = A_1 y(t-1) + \ldots + A_p y(t-p) + B_1 u(t-1) + \ldots + B_q u(t-q) + e(t) \quad (3)$$

where y is the observed BOLD signal, u is controlled stimuli input and e is noise.

Least-square estimation in ARX model

from (3)

$$\min_{A,B} \sum_{t=p+1}^{N} \left\| y(t) - \sum_{i=1}^{p} A_i y(t-i) - \sum_{i=1}^{q} B u(t-i) \right\|_2^2$$

$$\min_{A,B} \|Y - AH - BK\|_F^2 = \min_{A,B} \left\| Y - \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} H \\ K \end{bmatrix} \right\|_F^2$$

$$\theta = \begin{bmatrix} A & B \end{bmatrix}, \Lambda = \begin{bmatrix} H \\ K \end{bmatrix}$$

$$\min_{\theta} \|Y - \theta\Lambda\|_F^2$$

$$\theta = Y\Lambda^T \left(\Lambda\Lambda^T\right)^{-1}$$

Λ Full-rank

Least-square estimation in ARX model with zero constaints

$$\min_{A,B} \|Y - AH - BK\|_F^2$$
$$subject\ to\ (A_k)_{ij} = 0 \quad k = 1, 2, \ldots, p$$

$$\min_{x,z} \|y - Gx - Fz\|_2^2$$
$$subject\ to\ x_i = 0$$

$$\min_{\tilde{x},z} \left\| y - \tilde{G}\tilde{x} - Fz \right\|_2^2$$

$$\begin{bmatrix} \tilde{x} \\ z \end{bmatrix} = \begin{bmatrix} \tilde{G}^T \tilde{G} & \tilde{G}^T F \\ F^T \tilde{G} & F^T F \end{bmatrix} \begin{bmatrix} \tilde{G}^T \\ F^T \end{bmatrix} y$$

$\begin{bmatrix} \tilde{G} & F \end{bmatrix}$ Full-rank

fMRI data in project

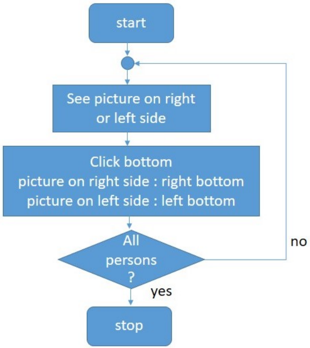


Figure: flow chart of visual-motor experiment

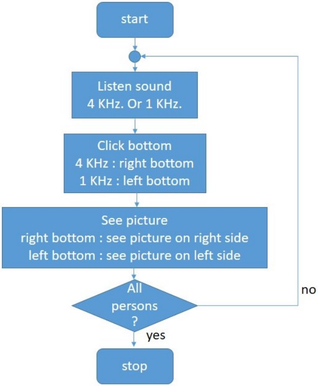


Figure: flow chart of motor-visual experiment

noise VS residual error (1)

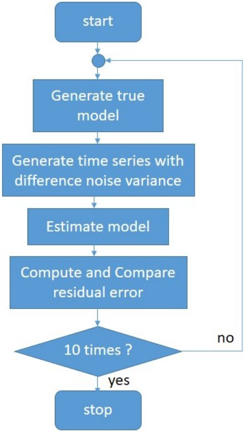


Figure: flow chart of noise VS residual error experiment

Hypothesis : If noise variance is increase, residual error will be increase.

noise VS residual error (2)

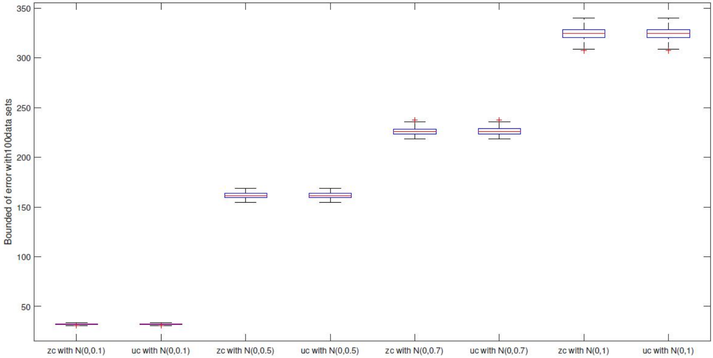


Figure: residual error form different noise, zc : we include true zero constraint in estimation, uc : we don't include zero constraint in estimation

Result : Noise variance is increase, residual error is increase.

numbers of data VS estimation error (1)

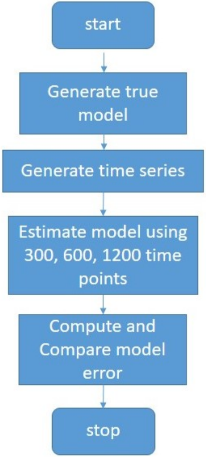


Figure: flow chart of numbers of data VS estimation error experiment

Hypothesis : If we use many time point data to estimate model, estimation error will be small.

numbers of data VS model error (2)

	error_300_data	error_600_data	error_1200_data
error_A_1	0.68619	0.2365	0.2365
error_A_2	1.2492	0.56347	0.23491
error_A_3	1.1646	0.60372	0.28544
error_A_4	0.40715	0.2945	0.1607
error_B_1	5.0978	5.1126	5.1102
error_B_2	3.8473	3.8529	3.8658
error_avg	2.0754	1.8038	1.6489

Table: estimation error form estimating by using different numbers of time point

Result : We use many time point data to estimate model, estimation error is small.

Model selection (1)

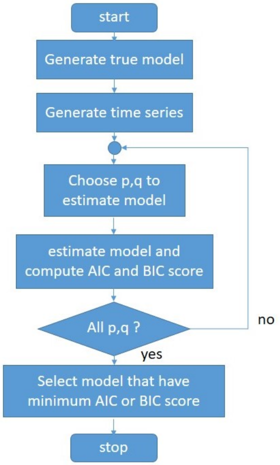
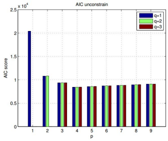


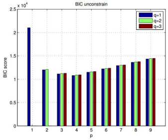
Figure: flow chart of model selection experiment

Hypothesis : AIC will choose model which is dense greater than BIC. BIC will choose model which is sparse greater than AIC

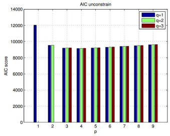
Model selection (2)



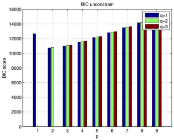
(a) AIC score for sparse model



(b) BIC score for sparse model



(c) AIC score for dense model



(d) BIC score for dense model

Result : AIC choose model which is dense greater than BIC. BIC choose model which is sparse greater than AIC. But no one choose model correctly.

Scope of work

- we focus only autoregressive model with exogenous input.
- we test model form generated data and real data.
- we compare model only data on motor-visual and visual-motor experiment.

Plan of working

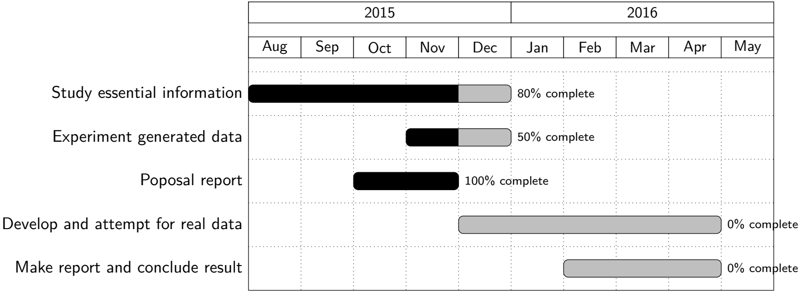


Figure: Gantt chart of the project

Expected outcomes

Expected outcomes

- Matlab code can solve estimating linear model problem of brain activity
- method to solve estimating autoregressive model with exogenous input to explain brain activity

Finish

- Matlab code that solve least square estimation with or without zero constrains

Q & A

References

F.-H. Lin, T. Witzel, T. Raij, J. Ahveninen, K. W.-K. Tsai, Y.-H. Chu, W.-T. Chang, A. Nummenmaa, J. R. Polimeni, W.-J. Kuo (2013)
fMRI hemodynamics accurately reflects neuronal timing in the human brain measured by MEG
Neuroimage 78, 372–384.

A. Pruttiakaravanich, J. Songsiri (2016)
A Review on dependence measures in exploring brain networks from fMRI data
Engineering Journal 20(3), 207–233.

Back up (Least-square estimation)

$$\underset{A,B}{\text{minimize}} \sum_{t=p+1}^{N} \|y(t) - \sum_{j=1}^{p} A_j y(t-j) - \sum_{j=1}^{q} B_k u(t-j)\|_2^2$$

$$\begin{aligned}
Y &= \begin{bmatrix} y(p+1) & y(p+2) & y(p+3) & \dots & y(N) \end{bmatrix} \\
A &= \begin{bmatrix} A_1 & A_2 & \dots & A_p \end{bmatrix} \\
B &= \begin{bmatrix} B_1 & B_2 & \dots & B_q \end{bmatrix} \\
H &= \begin{bmatrix} y(p) & y(p+1) & y(p+2) & \dots & y(N-1) \\ y(p-1) & y(p) & y(p+1) & \dots & y(N-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y(1) & y(2) & y(3) & \dots & y(N-p) \end{bmatrix} \\
K &= \begin{bmatrix} u(p) & u(p+1) & u(p+2) & \dots & u(N-1) \\ u(p-1) & u(p) & u(p+1) & \dots & u(N-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u(p-q+1) & u(p-q+2) & u(p-q+3) & \dots & u(N-q) \end{bmatrix}
\end{aligned}$$

Back up (Least-square estimation (2))

$$\underset{A,B}{\text{minimize}} \ \|Y - AH - BK\|_F^2$$

$$\theta = \begin{bmatrix} A & B \end{bmatrix}, L = \begin{bmatrix} H \\ K \end{bmatrix}$$

$$\frac{d}{d\theta} \|Y - \theta L\|_F^2 = -2(Y - \theta L)L^T = 0$$

$$\theta = YL^T(LL^T)^{-1}$$

$$\mathrm{AIC} = -2\mathcal{L} + 2d$$
$$\mathrm{BIC} = -2\mathcal{L} + d\log N$$
$$\mathcal{L}(\hat{A}, \hat{B}, \Sigma) = \frac{N-p}{2}\log\det\Sigma^{-1} - \frac{1}{2}\sum_{i=p+1}^{N}(y(i)-\hat{y}(i))^{T}\Sigma^{-1}(y(i)-\hat{y}(i))$$
$$\hat{y}(t) = \sum_{i=1}^{p}\hat{A}_i y(t-i) + \sum_{i=1}^{q}\hat{B}_i u(t-i)$$

$\mathcal{L}$ is log likelyhood function. d is a number of active parameters.