# A State-space model estimation of EEG signals using subspace identification

2102490 Project Proposal Year 2017

Satayu Chunnawong ID 5730569821

Advisor: Assist. Prof. Jitkomut Songsiri

Department of Electrical Engineering
Chulalongkorn University

# Outline

Introduction

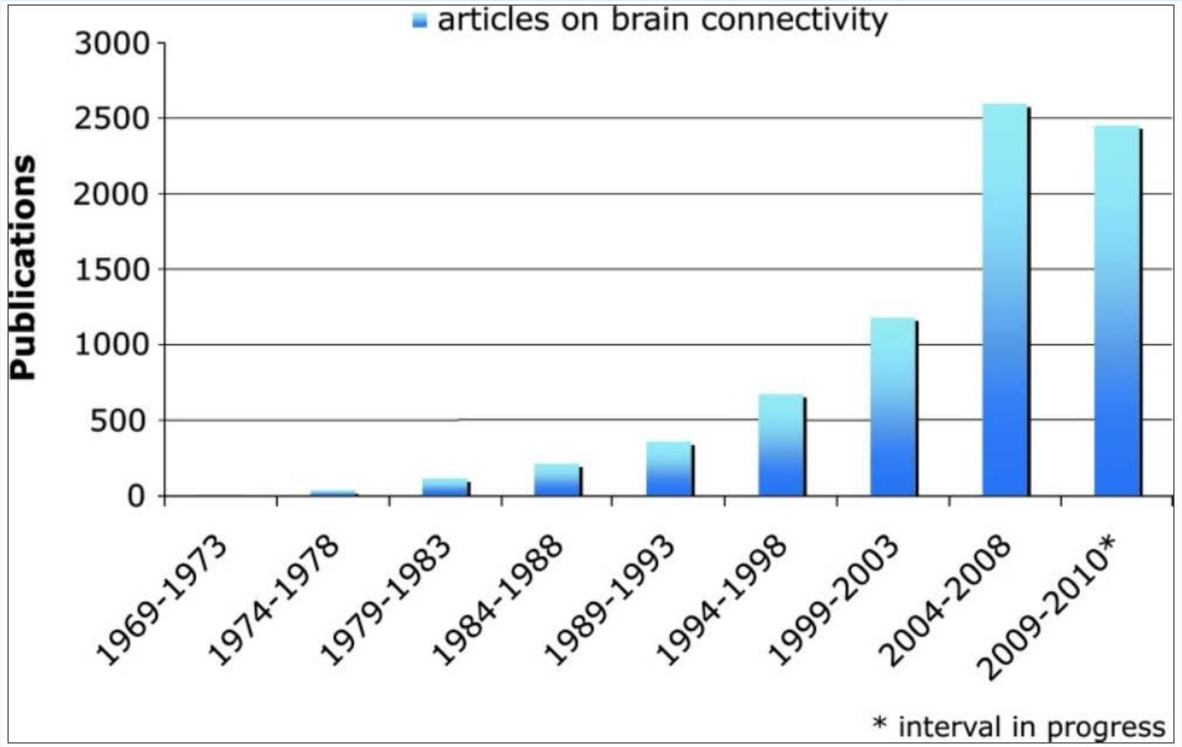
Background

Methodology

Preliminary Results

Project Overview

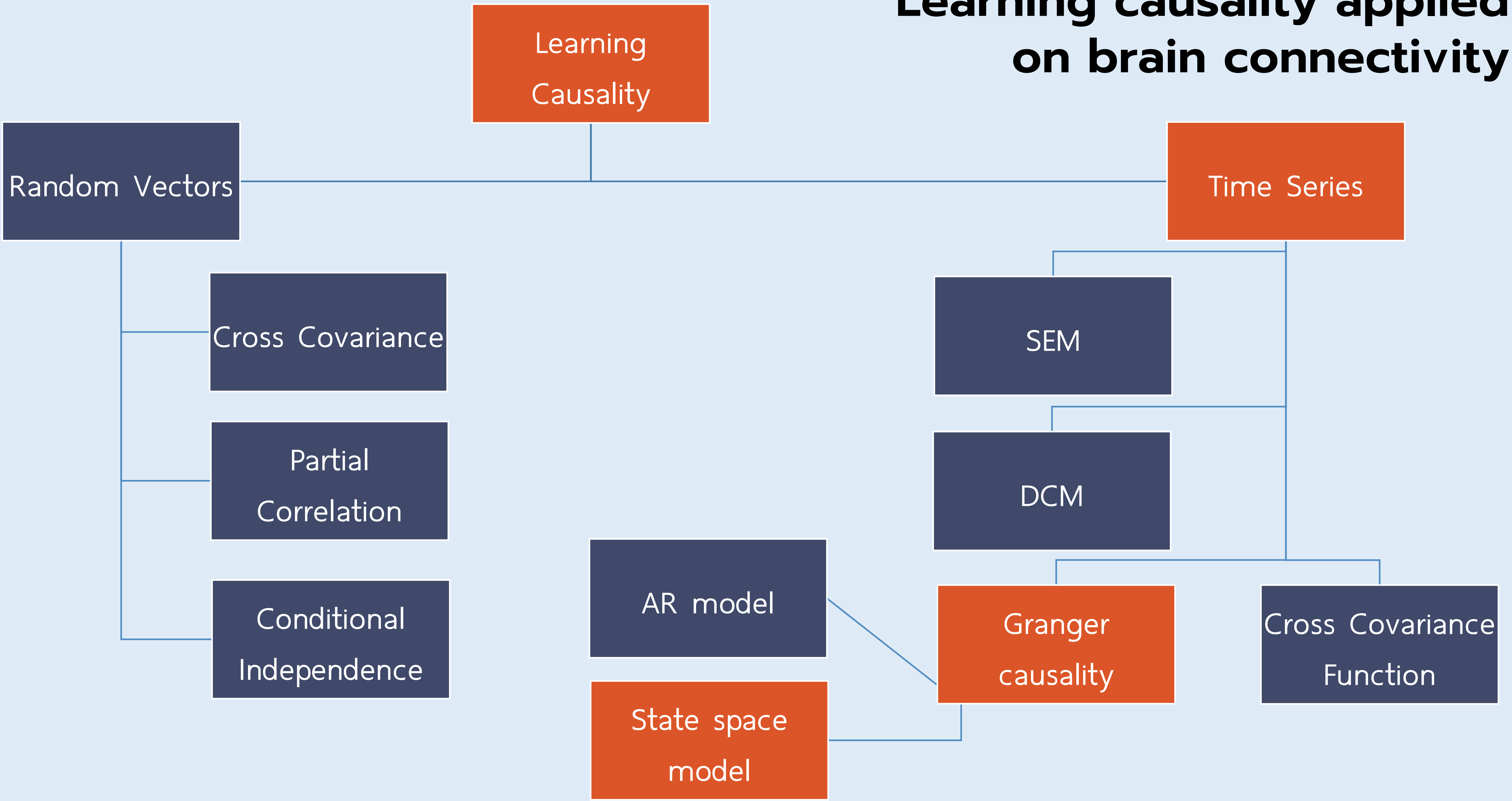
# Introduction

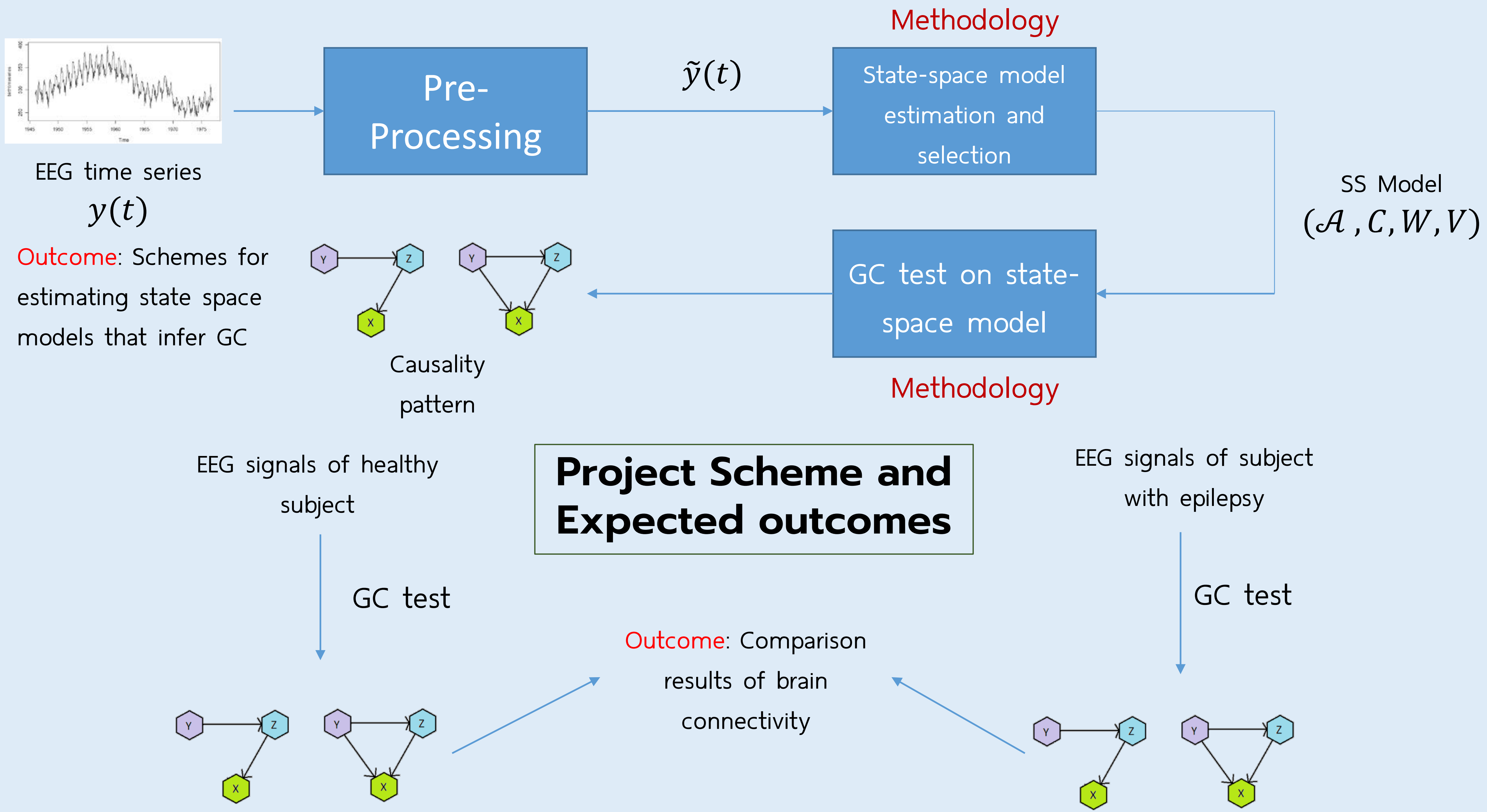



Source : C. Pawela and B. Biswal, "Brain Connectivity : A New Journal Emerges," *Brain Connectivity*, vol. 1, no. 2, 2011.

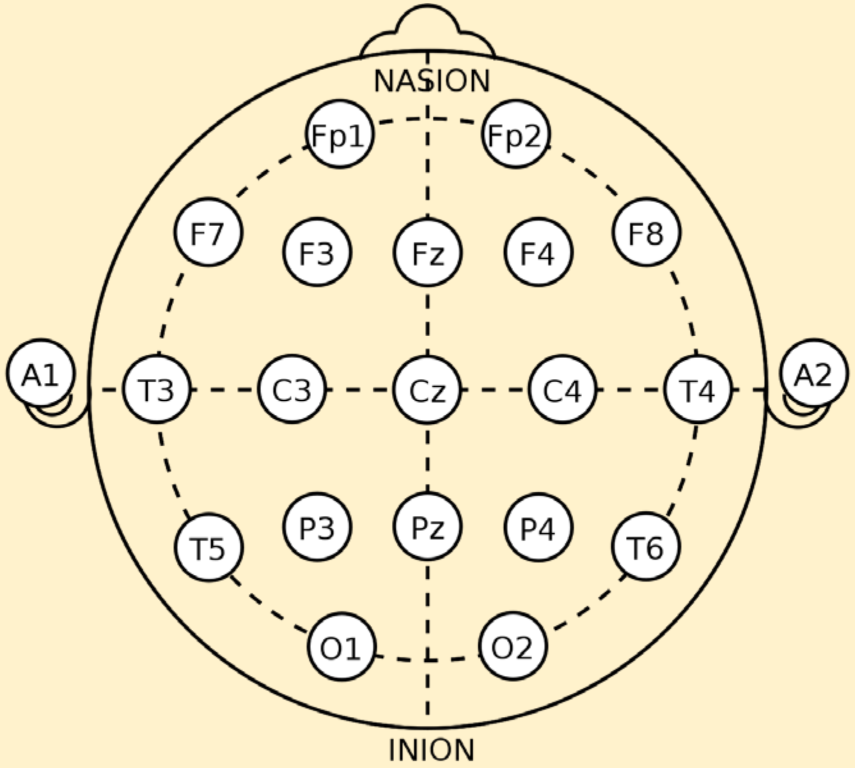
PubMed

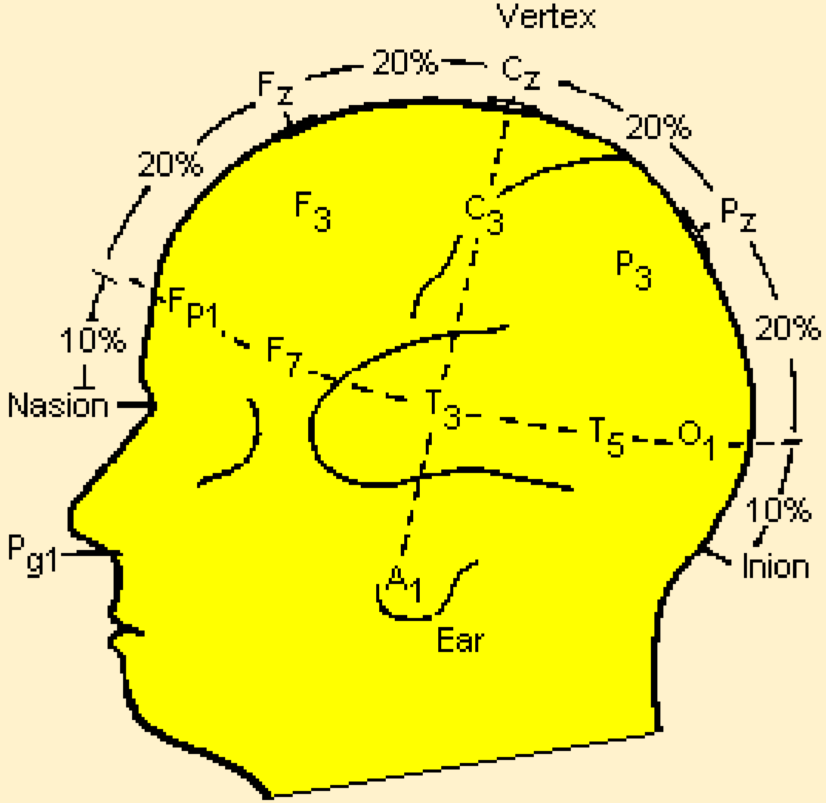
# Learning causality applied on brain connectivity



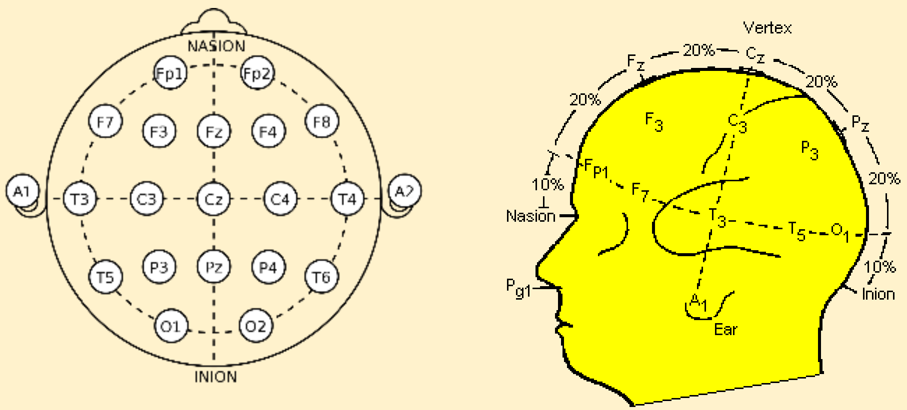

Methodology
Pre-Processing
$\tilde{y}(t)$
State-space model estimation and selection
EEG time series
$y(t)$
SS Model
$(\mathcal{A}, C, W, V)$
Outcome: Schemes for estimating state space models that infer GC
Y
Z
X
Causality pattern
GC test on state-space model
Methodology
EEG signals of healthy subject
Project Scheme and Expected outcomes
EEG signals of subject with epilepsy
GC test
GC test
Outcome: Comparison results of brain connectivity

# Background : EEG Models



There are many mathematical models that describe EEG signals. One can be generally described by linear Autoregressive (AR) model.

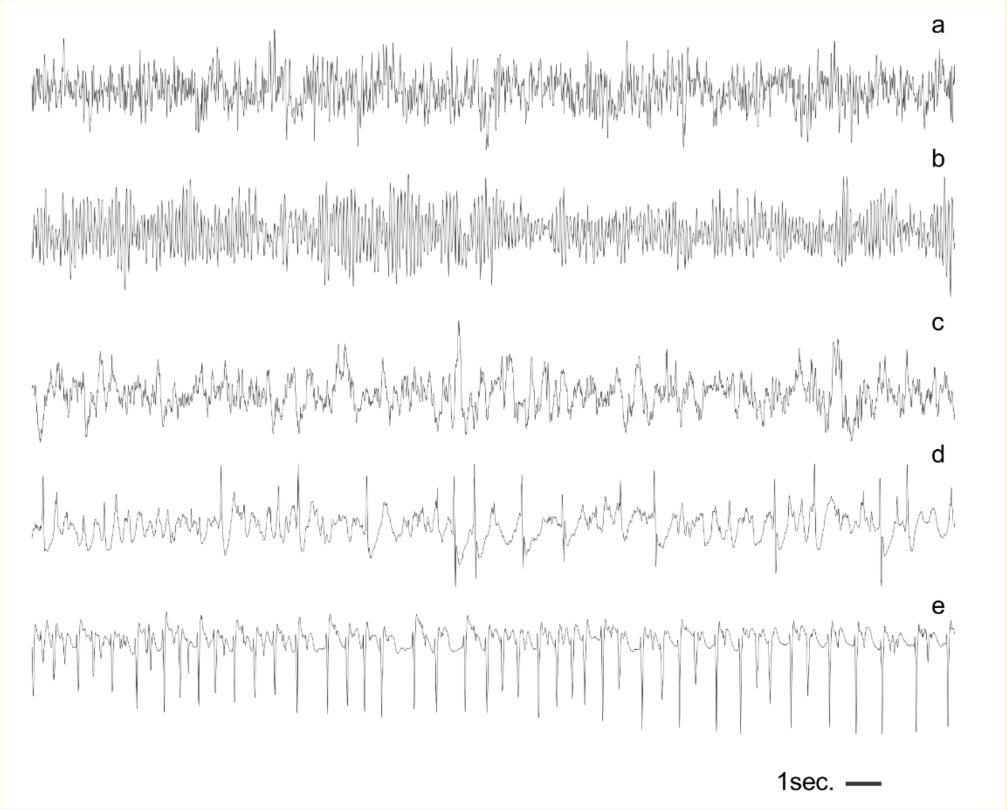



$$x(t) = \sum_{k=1}^{p} A_k x(t-k) + u(t)$$

$$y(t) = Lx(t) + v(t)$$

$x$ : Sources (not measured)

y : Measurement



In this project, we focus on only two Granger causality tests :

**GC test for AR model**

$$y(t) = \sum_{k=1}^{p} A_k y(t-k) + u(t)$$

According to Pruttiakaravanich (2016), If $y_j$ does not cause $y_i$, it can be shown that

$$(A_k)_{ij} = 0$$

## GC test for state space model (Seth, 2015)

$$z(t+1) = \mathcal{A}z(t) + w(t)$$
$$y(t) = Cz(t) + v(t) \qquad \text{(Full model)}$$
$$y^R(t) = C^R z(t) + v(t) \qquad \text{(Reduced model)}$$

To remove $y_j$ is to remove $j^{th}$ column of $C$

Let $\Sigma, \Sigma^R$ as prediction error covariance of full model and reduced model, respectively.

$$\mathcal{F}_{y_j \to y_i |\, all\ others\ y} = \log\left(\frac{\Sigma_{\mathrm{ii}}^R}{\Sigma_{\mathrm{ii}}}\right)$$

In general, prediction error of $y_i$ in reduced model is bigger prediction error of $y_i$ in full model. If $\Sigma_{\mathrm{ii}}^R = \Sigma_{\mathrm{ii}}$, it means $y_j$ does not cause $y_i$

# Methodology

## Stochastic Subspace Identification (Overschee and De Moor, 1996)

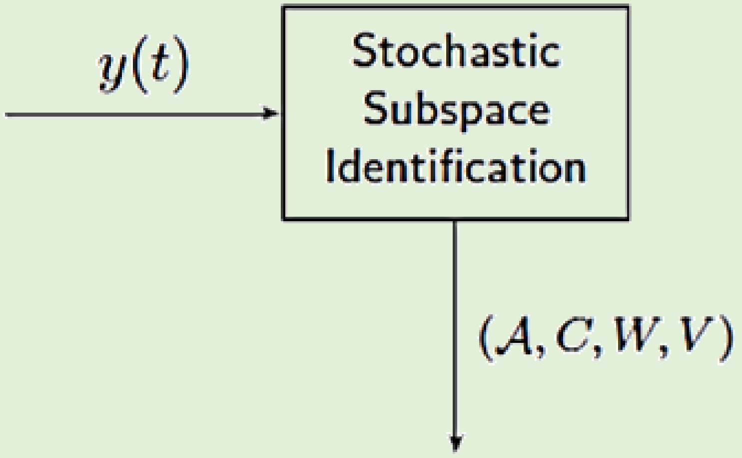
$$\mathcal{O}_i \triangleq Y_{i\,|\,2i-1}/Y_{0\,|\,i-1} = Y_f/Y_p$$

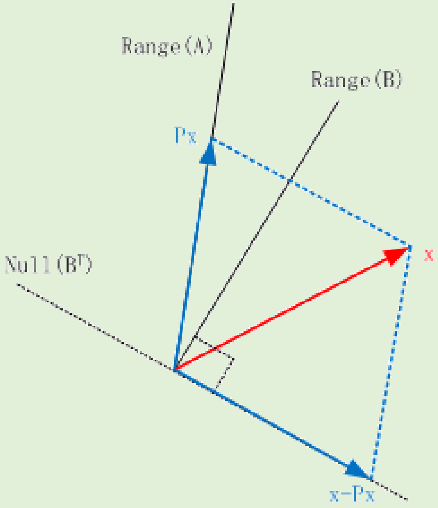
$$\hat{X}_i = \Gamma_i^{\dagger}\mathcal{O}_i$$

$$\begin{bmatrix} \hat{X}_{i+1} \\ Y_{i\,|i} \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} \hat{X}_i + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix}$$

$$\begin{bmatrix} \hat{A} \\ \hat{C} \end{bmatrix} = \begin{bmatrix} \hat{X}_{i+1} \\ Y_{i\,|\,i} \end{bmatrix} \hat{X}_i^{\dagger}$$

$$\begin{bmatrix} \hat{W} & \hat{S} \\ \hat{S}^T & \hat{V} \end{bmatrix} = (1/j) \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix} \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix}^T$$



# Methodology

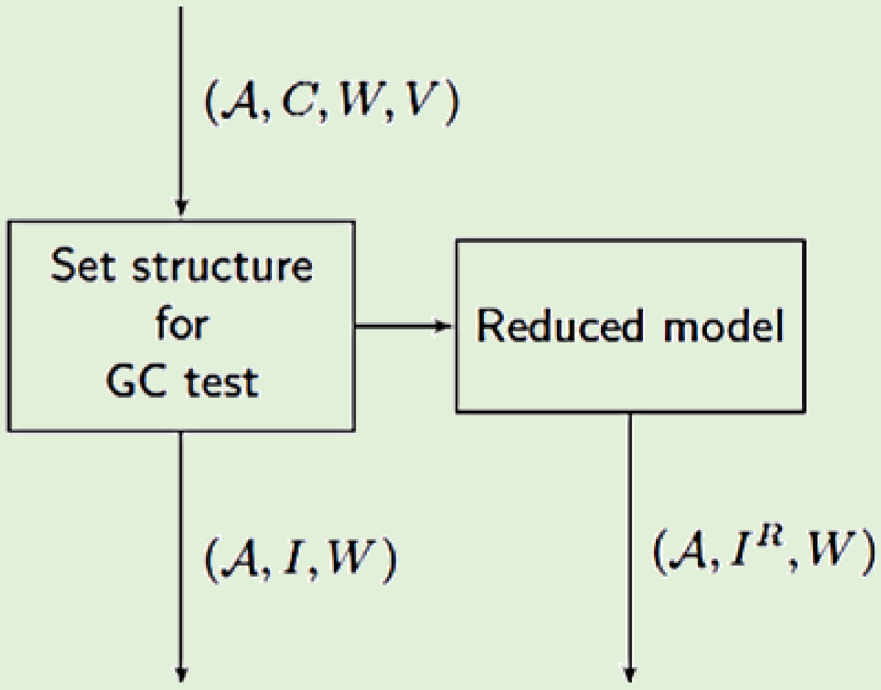
**Granger causality on State space model**

State space model (full model)

$$z(t+1) = \mathcal{A}z(t) + w(t)$$
$$y(t) = Cz(t) + v(t)$$

State space model (reduced model)

$$z(t+1) = \mathcal{A}z(t) + w(t)$$
$$y(t) = C^R z(t) + v(t)$$

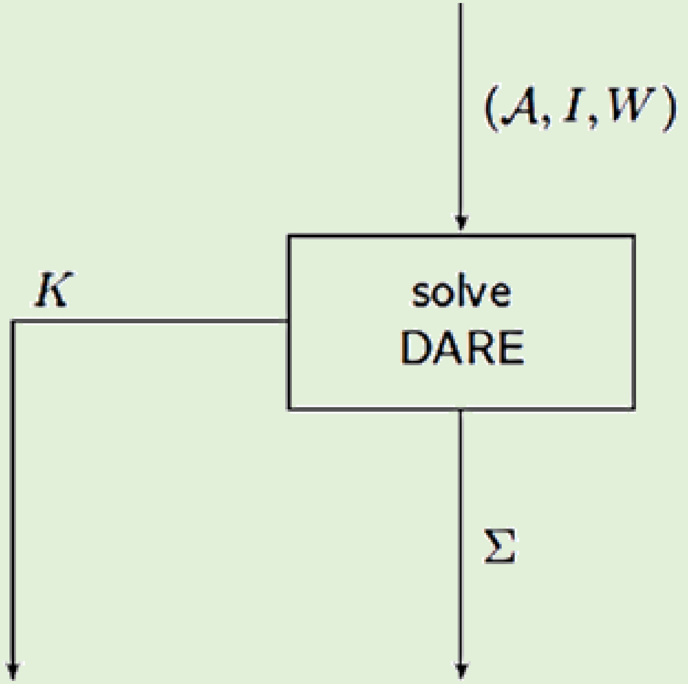



$\Sigma$: estimation error covariance of full model

$\Sigma^R$: estimation error covariance of reduced model

# Methodology

**Granger causality on State space model**




We find estimation error covariance :

$\Sigma = cov\,(z - \hat{z})$ for full model and $\Sigma^R$ for reduced model.

Where $\hat{z}$ is an optimal estimation of $z$ which observed by using **Kalman filter**

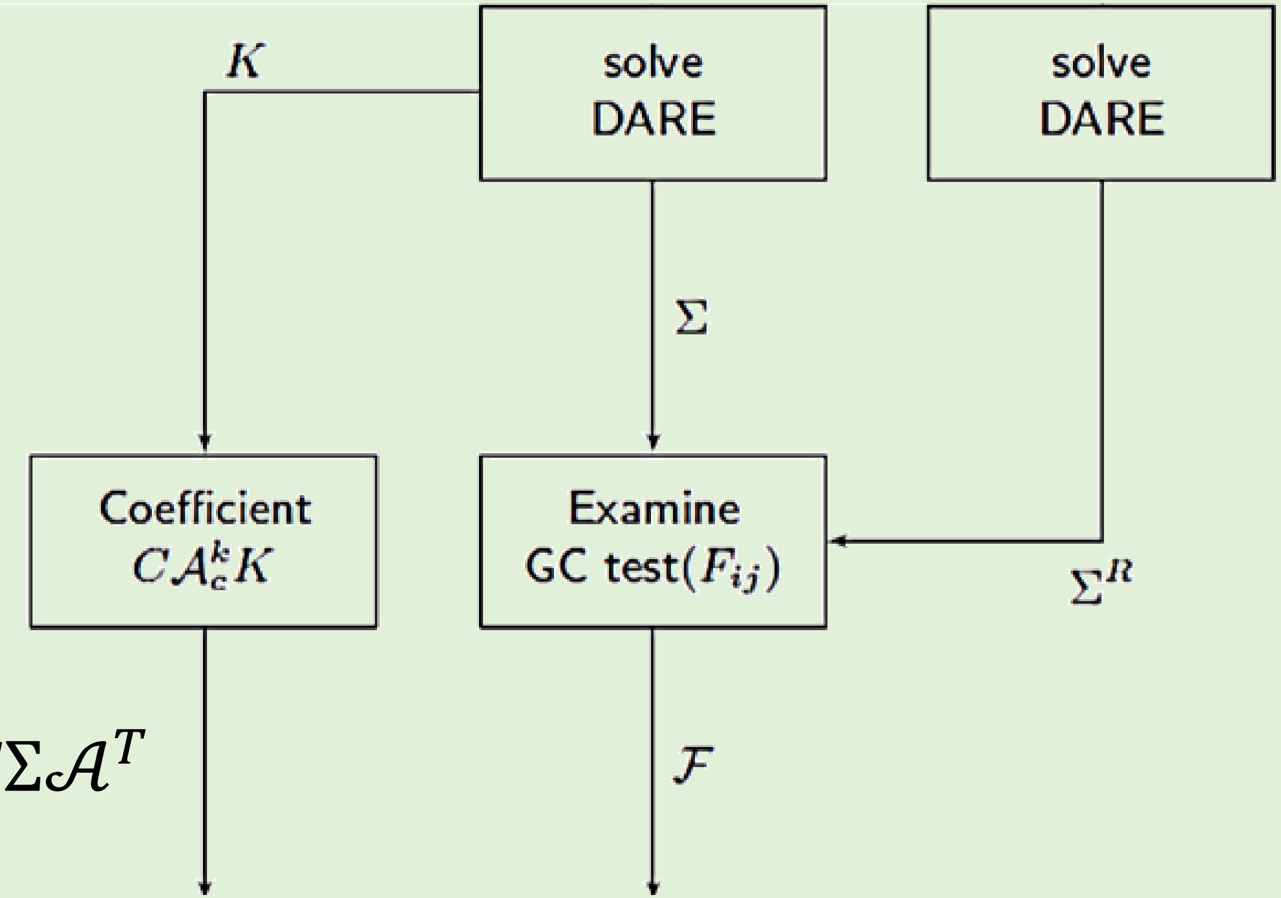
$$\hat{z}_{t+1|t} = \mathcal{A}\hat{z}_{t|t-1} + \Sigma_{t|t-1}C^T\left(C\Sigma_{t|t-1}C^T + R\right)^{-1}\left(y_t - C\hat{z}_{t|t-1}\right)$$
$$= \mathcal{A}\hat{z}_{t|t-1} + K(y_t - \hat{y}_{t|t-1})$$

# Methodology

## Granger causality on State space model

By assumption, we can solve steady state Kalman filter which satisfies DARE

$$\Sigma = \mathcal{A}\Sigma\mathcal{A}^T + W - \mathcal{A}\Sigma C^T(C\Sigma C^T)^{-1}C\Sigma\mathcal{A}^T$$




Then, Seth (2015) suggest to determine the time-domain Granger causality shown as

$$\mathcal{F}_{x_j \to x_i | \, all \; others \; x} = \log\left(\frac{\Sigma_{ii}^R}{\Sigma_{\mathrm{ii}}}\right)$$

# Preliminary Results

There are two experiments in this semester

Equivalence of GC test on AR and state space model

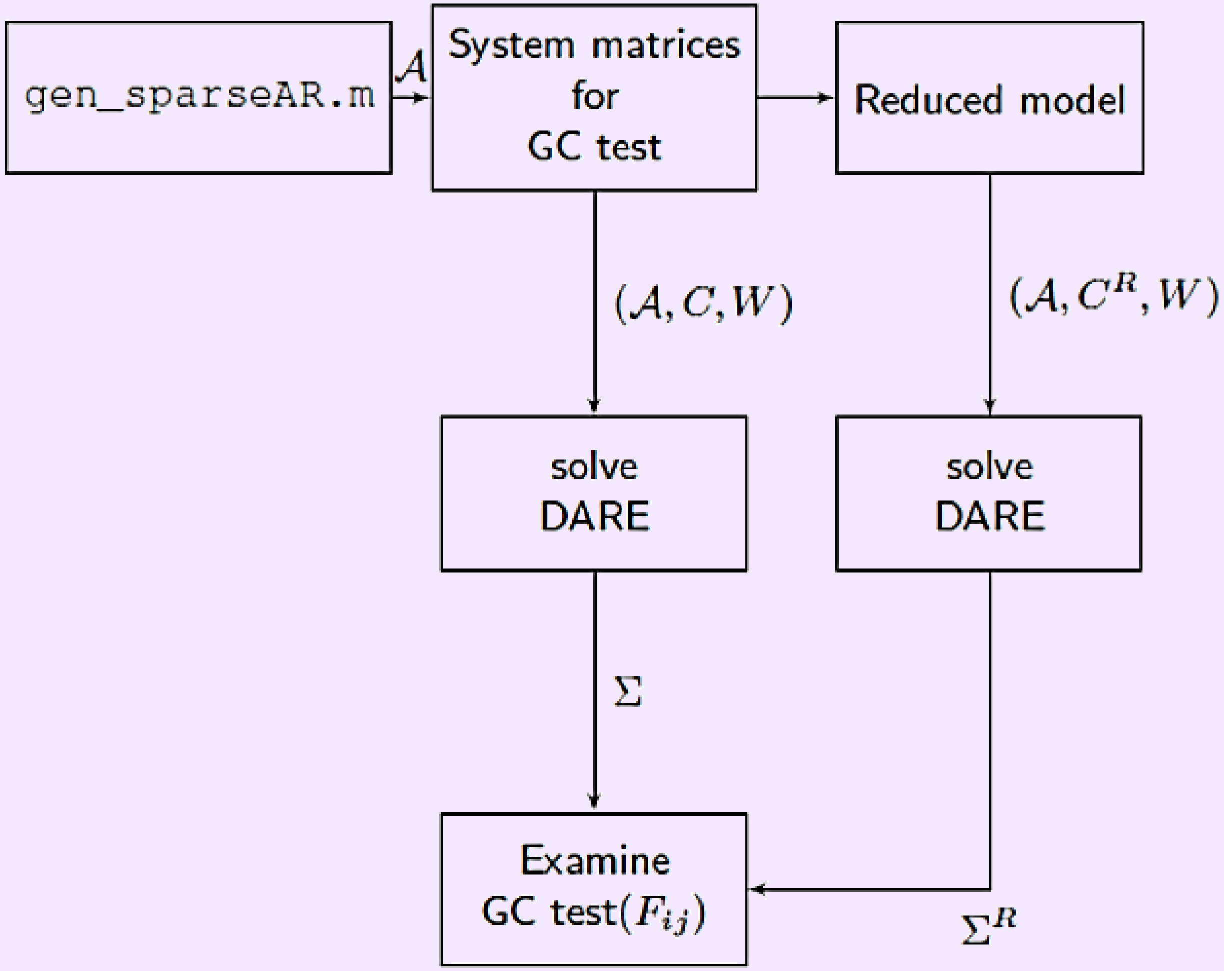
Hypothesis: If ground truth is AR, the result of GC test on state space model is the same result as GC test on AR model

GC test on estimated state space model

Hypothesis: If EEG signals are generated based on AR model, the result of GC test on estimated state space model is the same result as GC test on AR model

# Preliminary Results

Equivalence of GC test



# Preliminary Results

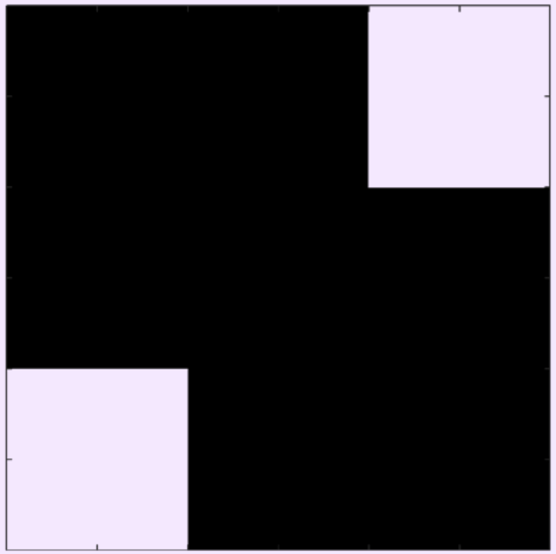
We format state space model from ground truth AR model

$$z(t+1) = \mathcal{A}z(t) + w(t)$$
$$y(t) = Cz(t) + \epsilon(t)$$

$$\mathcal{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_p \\ I & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix} \quad C = \begin{bmatrix} L^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \quad w(t) = \begin{bmatrix} u(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \epsilon(t) = \begin{bmatrix} v(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

# Preliminary Results

sparsity of AR coefficients



```
>> F
```

**GC test on AR model**

```
F =

0.5207 0.0016 0.0000
0.2412 0.4073 0.1417
0.0000 0.0282 0.1385
```

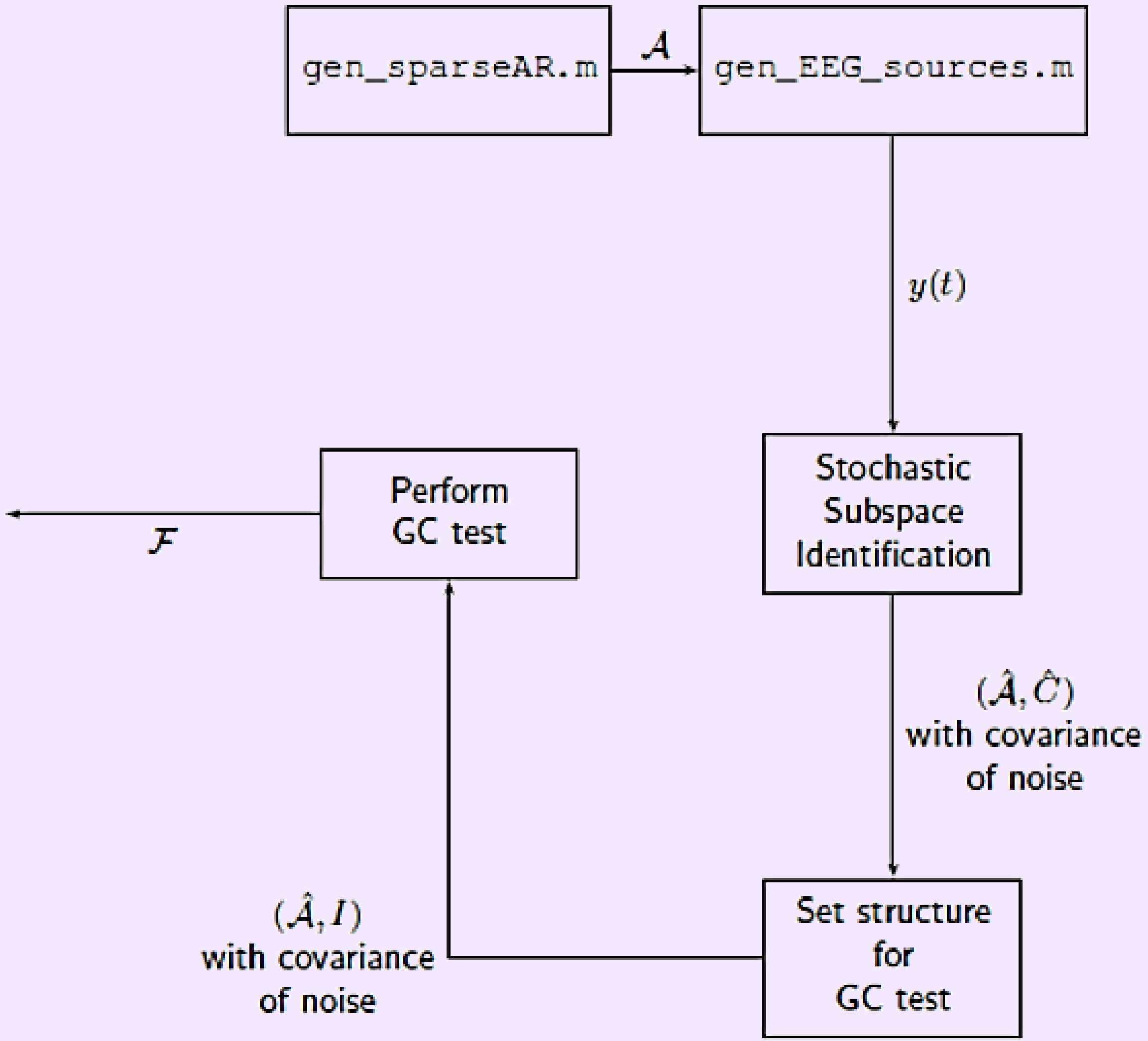
```
>> F_r
```

**GC test on state space model**

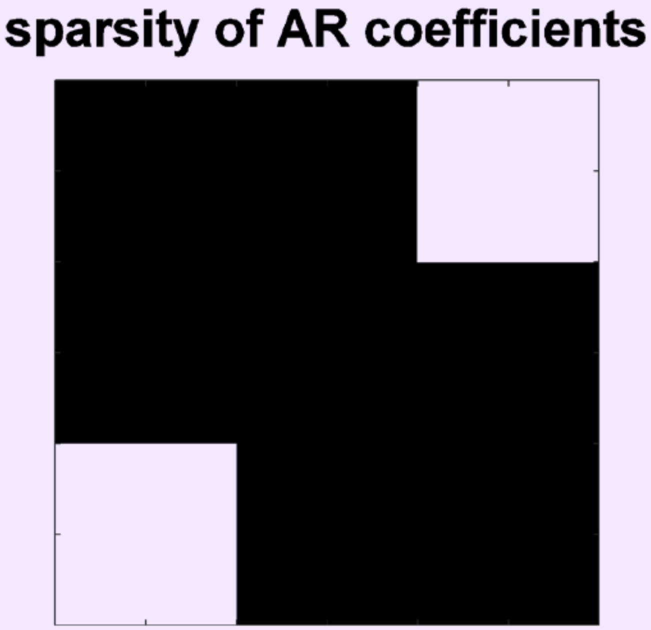
```
F_r =

0.5207 0.0016 0.0000
0.2412 0.4073 0.1417
0.0000 0.0282 0.1385
```

# Preliminary Results

**GC test on estimated state space model**



# Preliminary Results




```
>> F
```

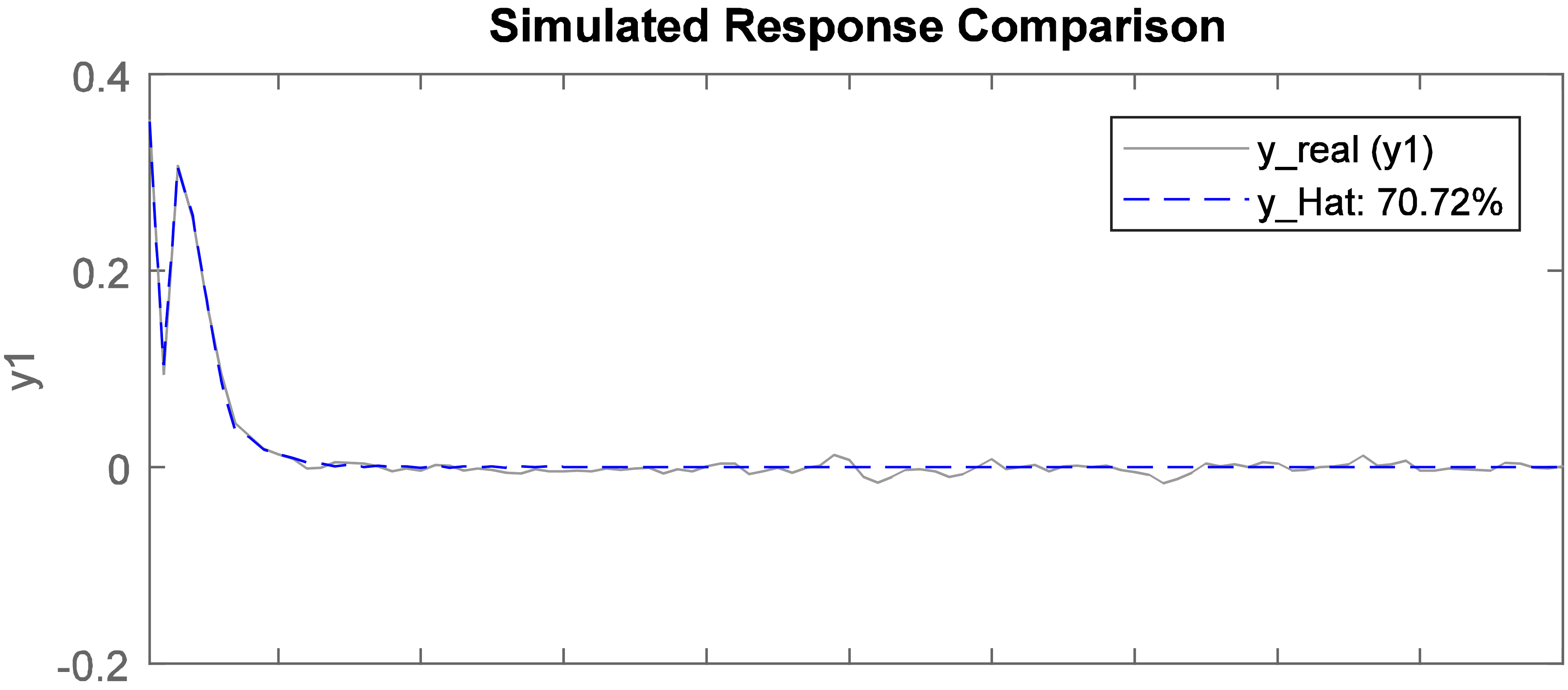
**GC test on AR model**

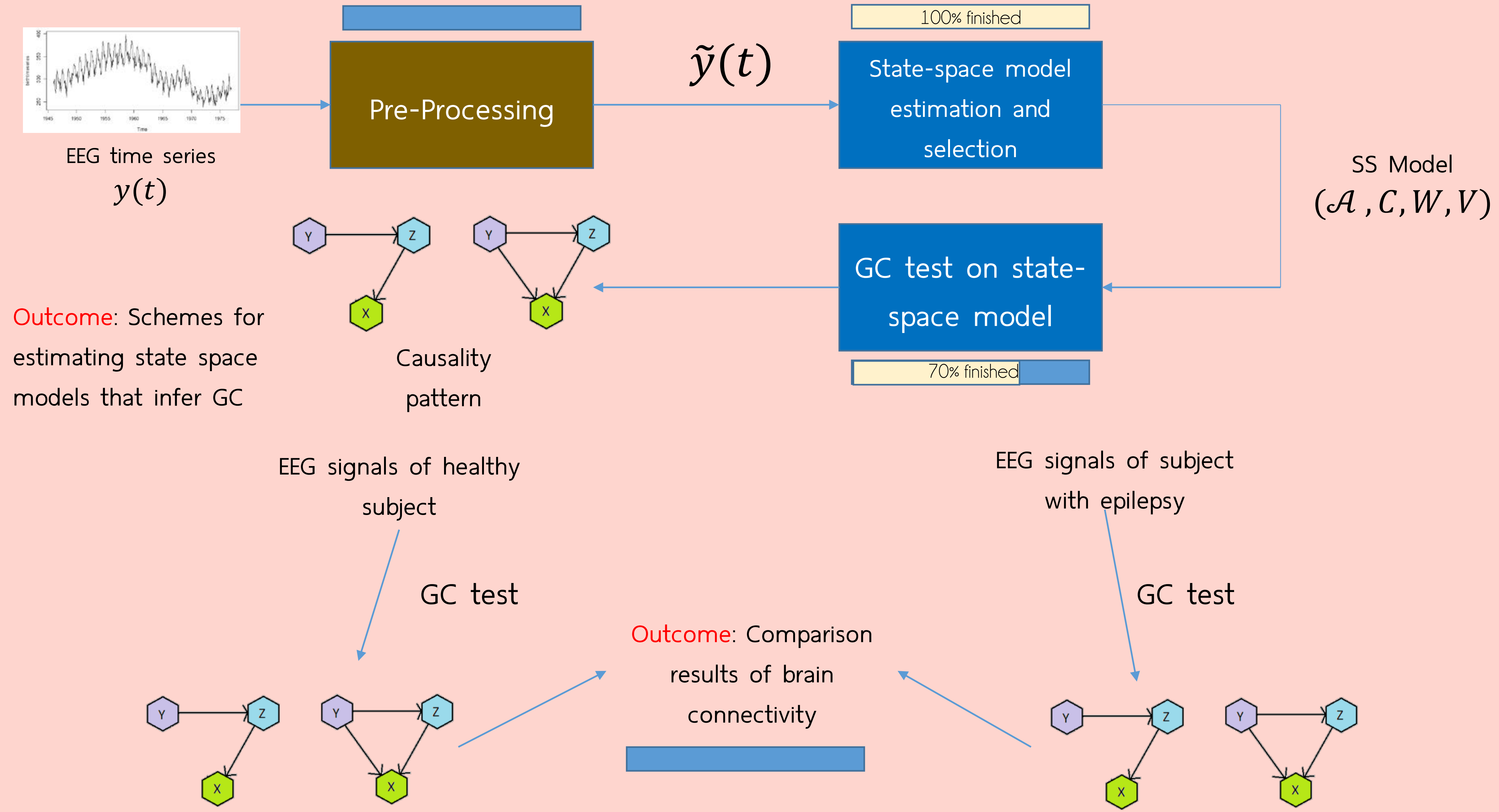
```
F =

0.5207 0.0016 0.0000
0.2412 0.4073 0.1417
0.0000 0.0282 0.1385
```

```
>> F_ss
```

**GC test on estimated state space model**

```
F_ss =

0.2162 0.2033 0.6843
0.2532 3.1890 0.7549
0.2575 0.9480 1.4804
```


Simulated Response Comparison
0.4
0.2
0
-0.2
y1
y_real (y1)
y_Hat: 70.72%



EEG time series
$y(t)$
Pre-Processing
$\tilde{y}(t)$
100% finished
State-space model estimation and selection
SS Model
$(\mathcal{A}, C, W, V)$
GC test on state-space model
70% finished
Y
Z
X
Causality pattern
Outcome: Schemes for estimating state space models that infer GC
EEG signals of healthy subject
EEG signals of subject with epilepsy
GC test
GC test
Outcome: Comparison results of brain connectivity

# Q&A

# Reference


- L. Barnett and A. K. Seth, "Granger causality for state space models," *Physical Review E*, vol. 91, no. 4, 2015.
- A. Pruttiakaravanich and J. Songsiri, "A review on dependence measures in exploring brain networks from fMRI data," *Engineering Journal*, vol. 20, no. 3, pp. 207–233, 2016
- P. van Overschee and B. de Moor, *Subspace Identification for Linear Systems*. Hoboken, New Jersey: Wiley, 1996

# Backup

## $C\mathcal{A}_c^k K$ coefficeints

We showed that the coefficient from GC test on AR model have same structure to the coefficient from GC test on state space based on ground truth AR model

Let $K$ as gain solved from steady state Kalman filter

$$K = \mathcal{A}\Sigma C^T(C\Sigma C^T + V)^{-1}$$
$$= \mathcal{A}\begin{bmatrix}\Sigma_{11}^T & \Sigma_{12}^T & \dots & \Sigma_{1p}^T\end{bmatrix}^T \Sigma_{11}^{-1}$$

Because the solution of DARE remains only $\Sigma_{11}$

$$K = \mathcal{A}[I \quad 0 \quad 0 \quad 0]^T$$
$$= [A_1^T \quad I \quad 0 \quad 0]^T$$

# Backup

$C\mathcal{A}_c^k K$ coefficeints

Given state observer gain $\mathcal{A}_c = \mathcal{A} - KC$ , we have

$$\mathcal{A}_c = \begin{bmatrix} 0 & A_2 & \cdots & A_p \\ 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix}$$

Then, multiply by $C$ on the left hand side and $K$ on the right hand side

$$C\mathcal{A}_c^k K = [I \quad 0 \quad \cdots \quad 0] \begin{bmatrix} 0 & A_2 & \cdots & A_p \\ 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix}^{\mathrm{k}} [A_1^T \quad I \quad 0 \quad 0]^T$$

# Backup

$C\mathcal{A}_c^k K$ coefficeints

$$C\mathcal{A}_c^k K = [I \quad 0 \quad \cdots \quad 0] \begin{bmatrix} 0 & A_2 & \cdots & A_p \\ 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix}^{\mathrm{k}} [A_1^T \quad I \quad 0 \quad 0]^T$$

The result showed that

When $k = 0$ $\quad CK = A_1$

When $k = 1$ $\quad C\mathcal{A}_c K = A_2$

When $k = 2$ $\quad C\mathcal{A}_c^2 K = A_3$

$\vdots$

When $k = p - 1$ $\quad C\mathcal{A}_c^{p-1} K = A_p$