

# Recursive Estimation of Solar forecasting at Chulalongkorn University

Name : Tony Fang

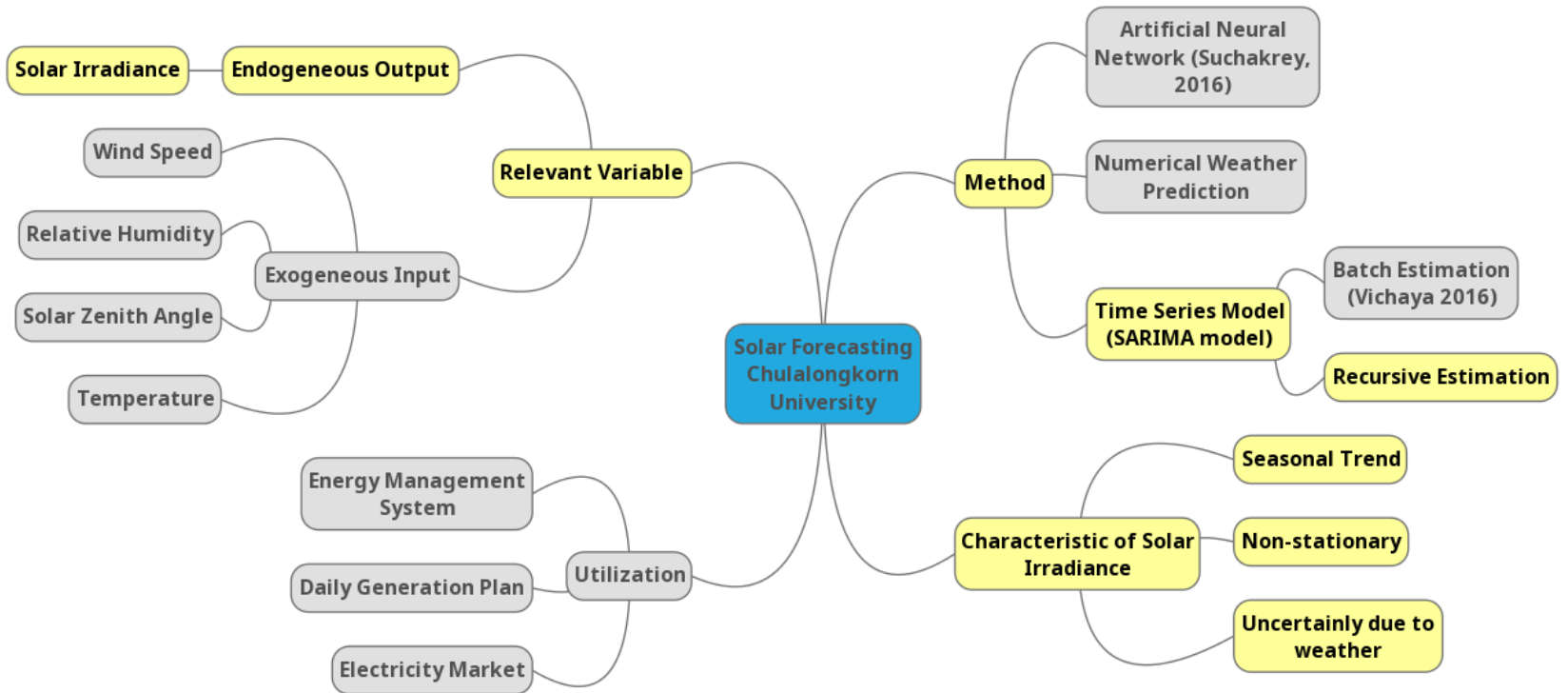
ID : 5730212721

Advisor : Assist. Prof. Jitkomut Songsiri

# Outline

- Introduction
- Objective
- Method & Result
- Conclusion

# Introduction



# Objective

- Apply recursive estimation of parameter on SARIMA model for solar irradiance forecasting at Chulalongkorn University.
- Apply recursive SARIMA model on EE-CU solar power forecasting (SPF) server.

# SARIMA model

$$\tilde{A}(L^T)(1 - L^T)^D A(L)(1 - L)^d I(t) = \tilde{C}(L^T)C(L)e(t)$$

$$\tilde{A}(L^T) = 1 - (\tilde{A}_1 L^T + \tilde{A}_2 L^{2T} + \dots + \tilde{A}_P L^{PT})$$

$$A(L) = 1 - (A_1 L + A_2 L^2 + \dots + A_p L^p)$$

$$\tilde{C}(L^T) = 1 + (\tilde{C}_1 L^T + \tilde{C}_2 L^{2T} + \dots + \tilde{C}_Q L^{QT})$$

$$C(L) = 1 + (C_1 L + C_2 L^2 + \dots + C_q L^q)$$

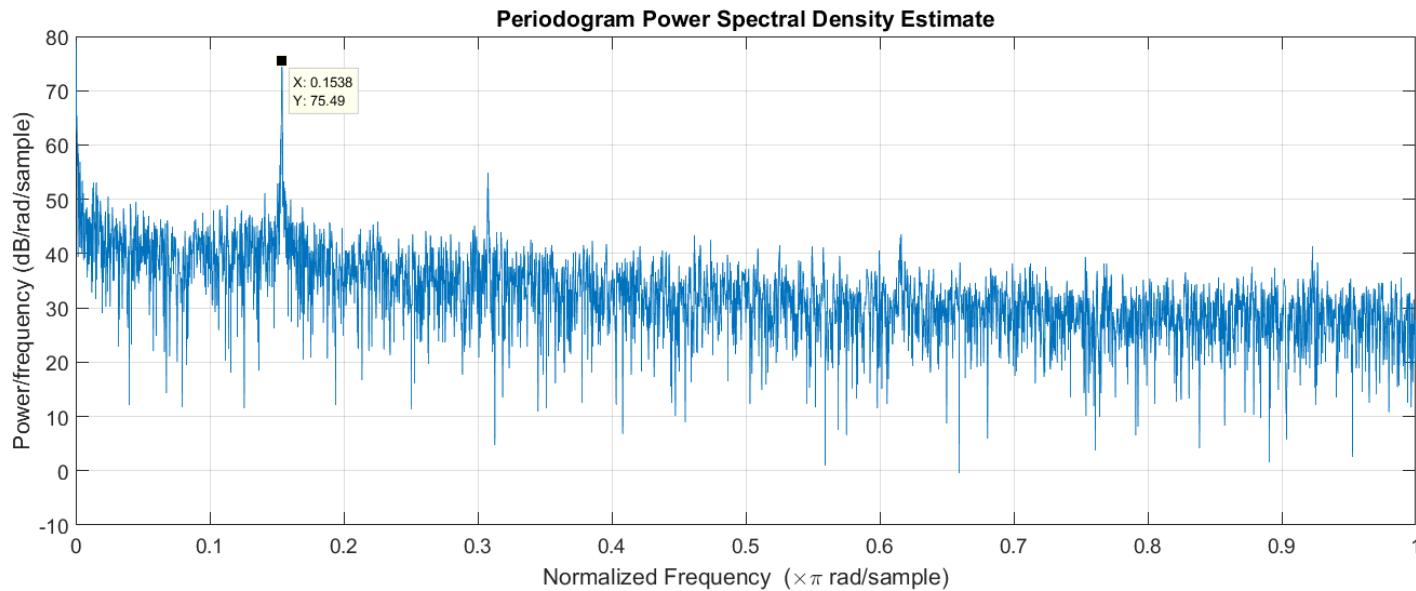
Written as  $SARIMA(p, d, q)(P, D, Q)_T$

# Method & Result

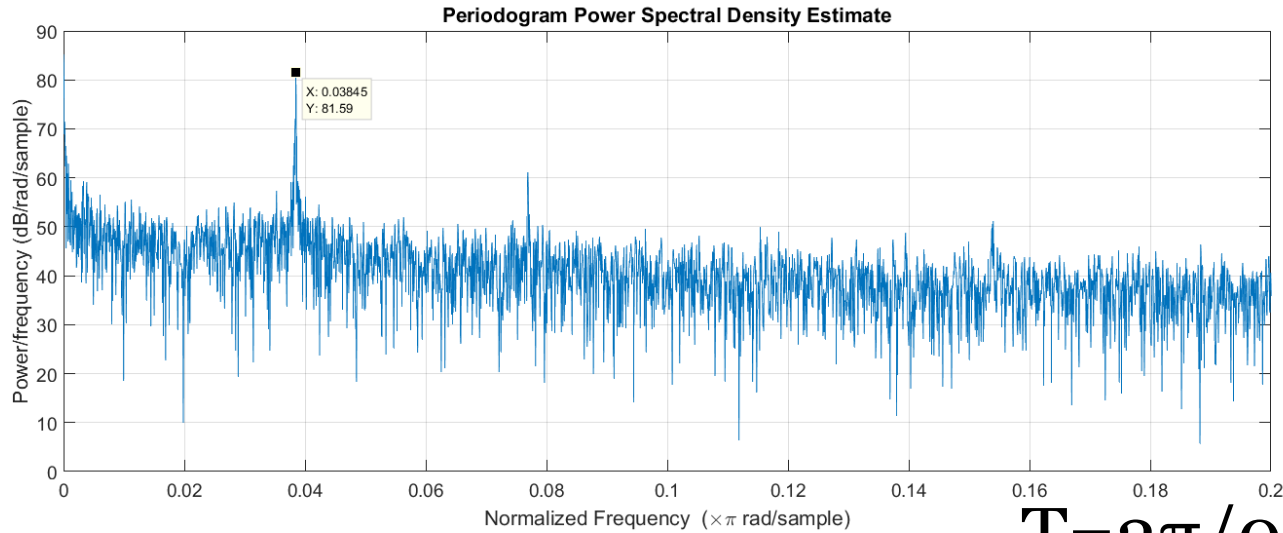
- Find seasonal trend
- Differencing
- ML and Model Validation
- Forecasting

# Finding Seasonal Trend

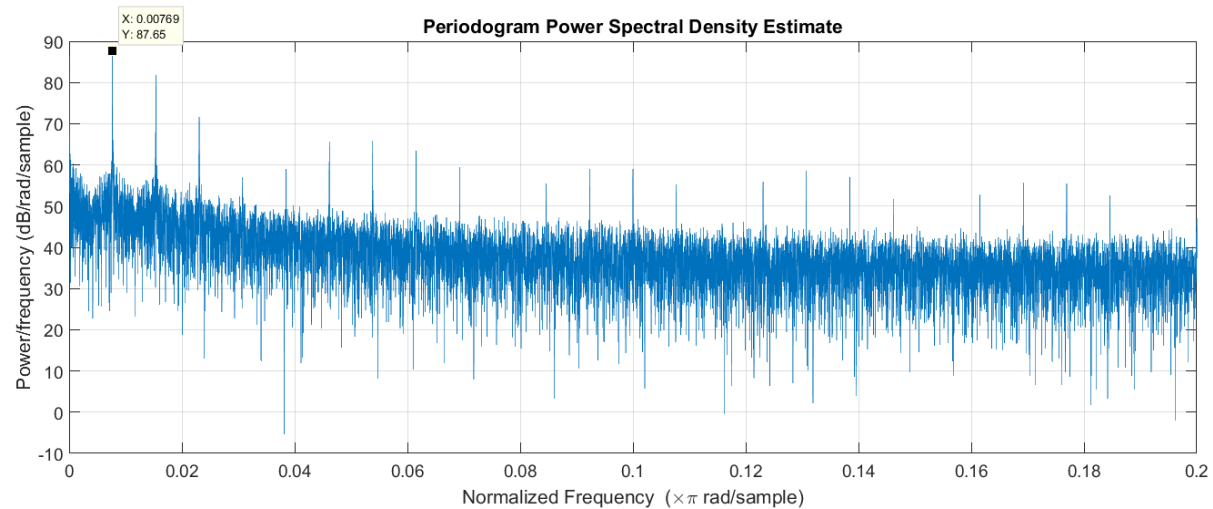
Using Fast Fourier Transform to find the power spectral density(PSD).  
Only high energy frequency and label them  $\omega_i$



$$T = 2\pi / 0.1538 \pi \approx 13$$



$$T = 2\pi / 0.03845 \pi \approx 52$$

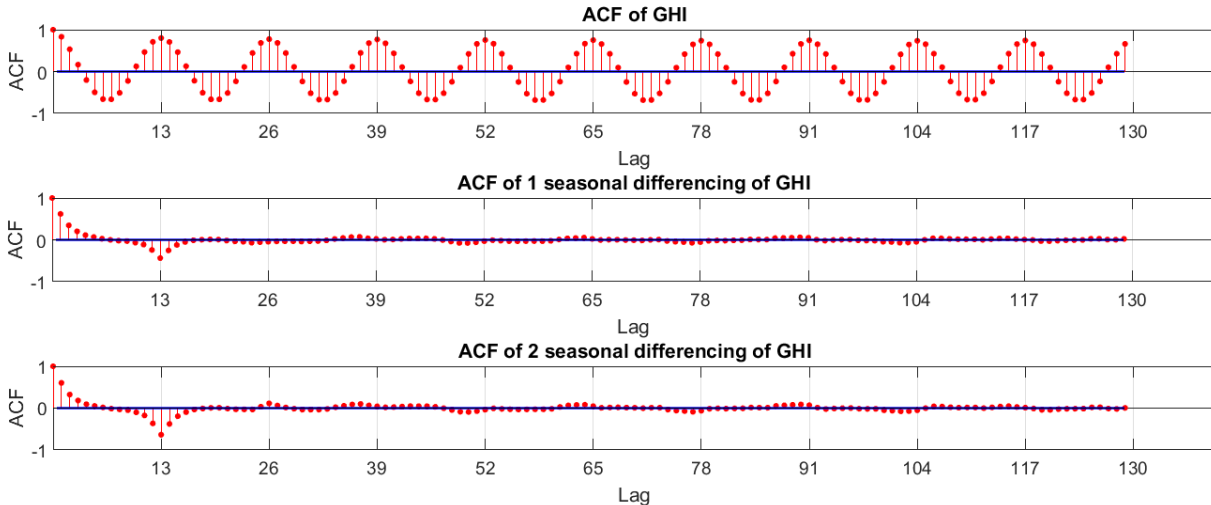


$$T = 2\pi / 0.00769 \pi \approx 260$$



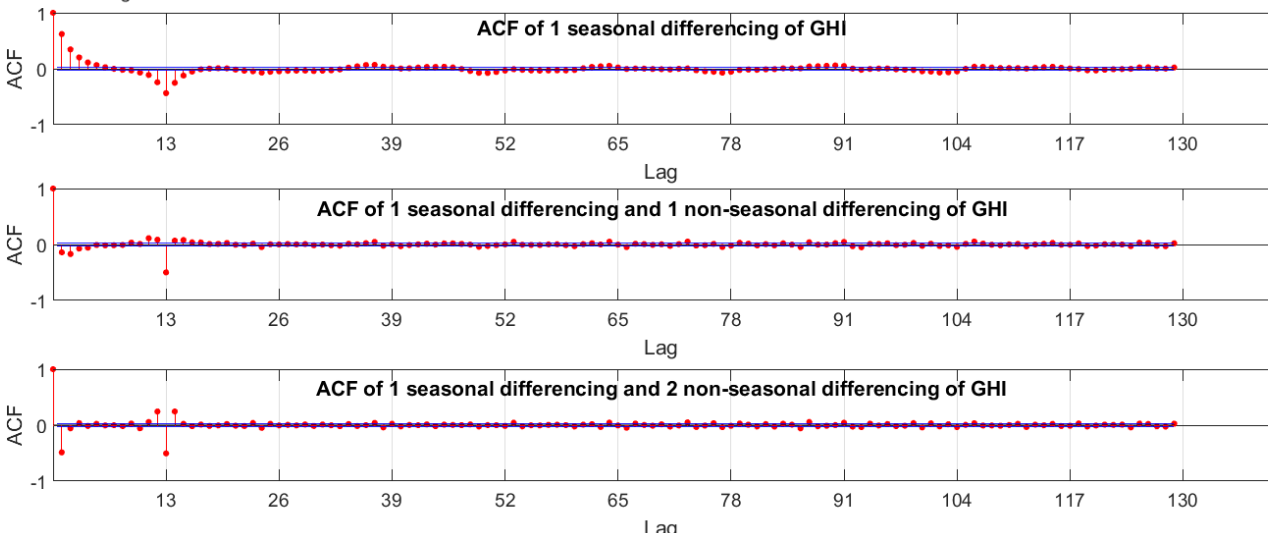
# Differencing

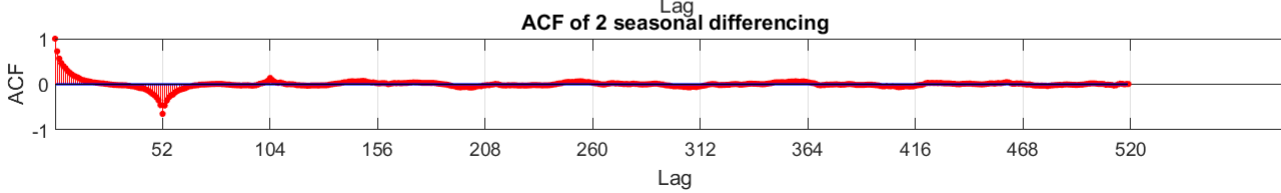
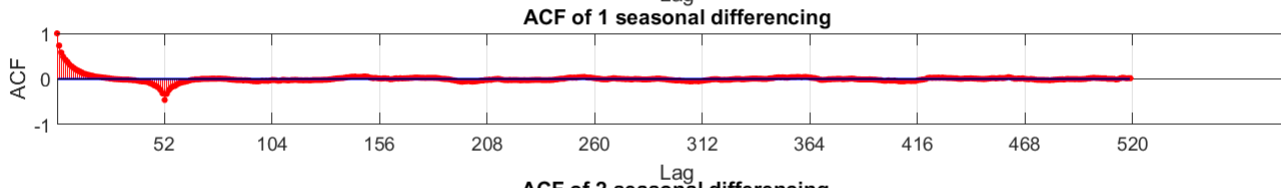
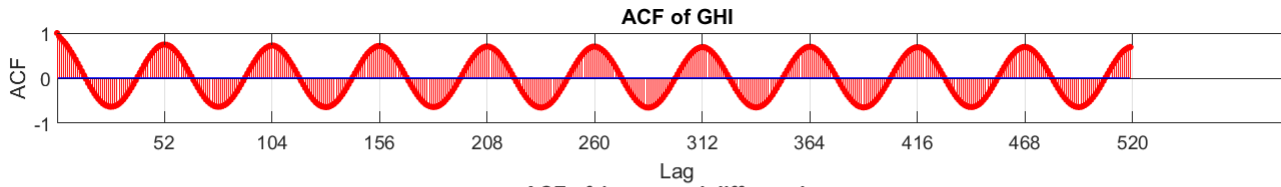
Apply Autocorrelation Function(ACF) and Partial Autocorrelation Function(PACF) finding integrated order  $d$  and  $D$



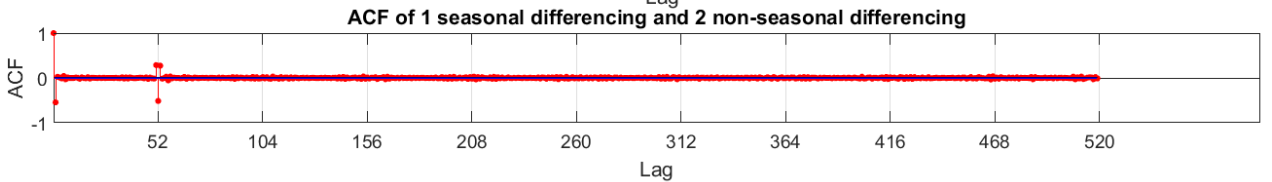
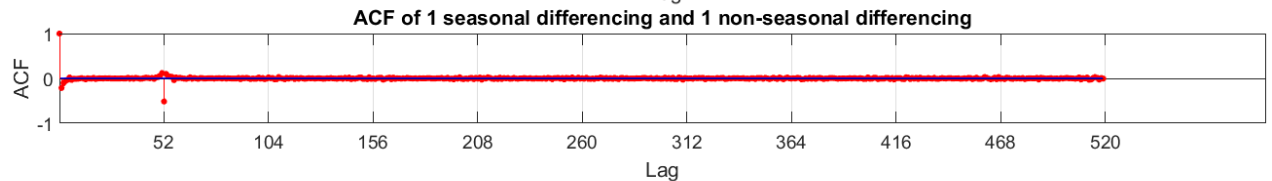
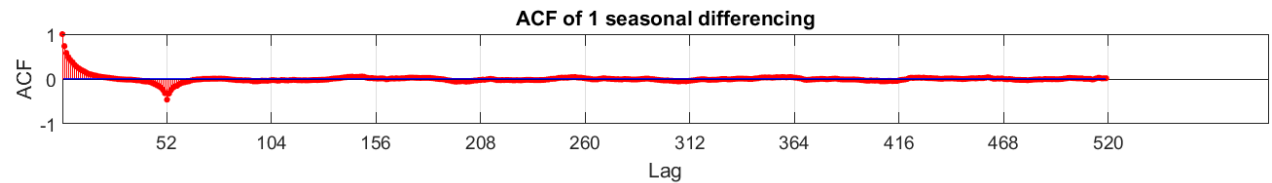
Choose  
 $D=1$

Choose  
 $d=1$

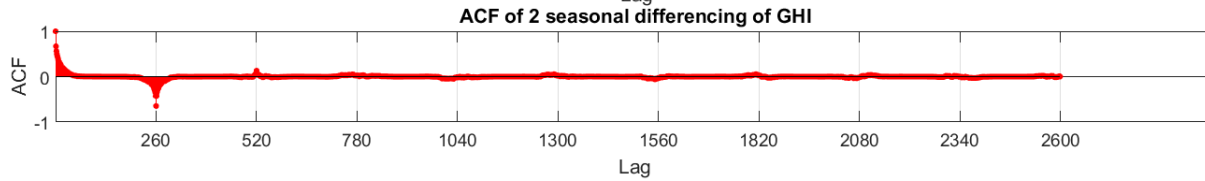
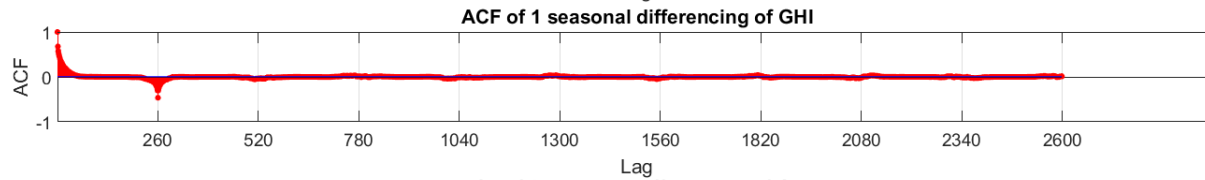
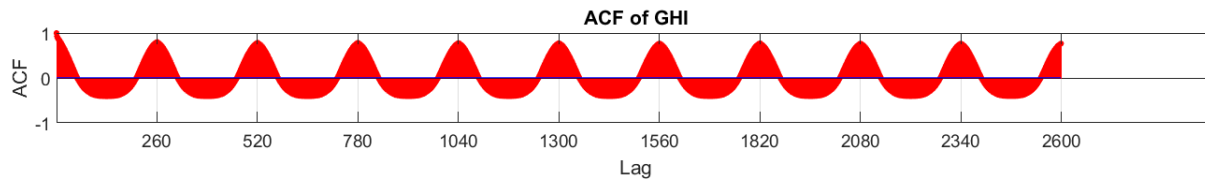




Choose  
 $D=1$

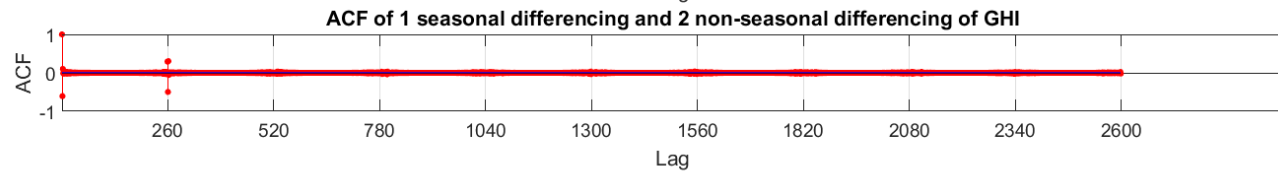
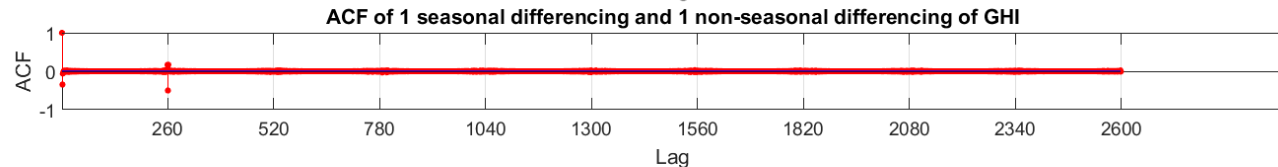
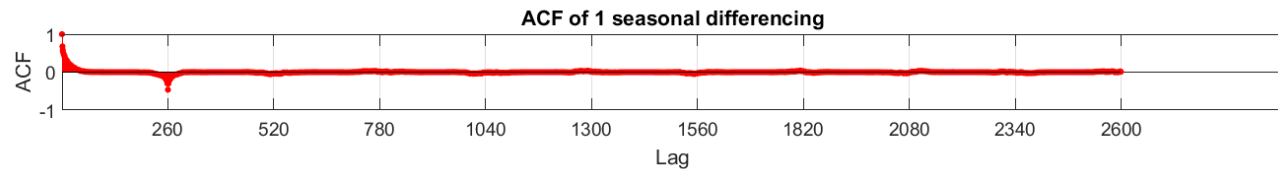


Choose  
 $d=1$



Choose  
D=1

Choose  
d=1



# Range of order to choose model

	<b>60 minutes</b>	<b>15 minutes</b>	<b>3 minutes</b>
AR(p)	0-8	0-8	0-5
MA(q)	0-4	0-4	0-4
SAR(P)	0-4	0-6	0-2
SMA(Q)	0-1	0-1	0-1

# Maximum Likelihood Estimation

$$\hat{\theta}_{ML} = \max_{\theta} -\frac{N-p}{2} \ln(2\pi) - \frac{N-p}{2} \ln(\sigma^2) - \sum_{t=p+1}^N \frac{v(t)^2}{2\sigma^2}$$

where,

$$\theta = [A_1 \ A_2 \ \dots \ A_p \ C_1 \ C_2 \ \dots \ C_q]$$

$$\begin{bmatrix} v(p+1) \\ v(p+2) \\ \vdots \\ v(N) \end{bmatrix}$$

$$= \begin{bmatrix} y(p+1) \\ y(p+2) \\ \vdots \\ y(N) \end{bmatrix} - \begin{bmatrix} y(p) & y(p-1) & \dots & y(1) & v(p) & v(p-1) & \dots & v(p-q+1) \\ y(p+1) & y(p) & \dots & y(2) & v(p+1) & v(p) & \dots & v(p-q+2) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ y(N-1) & y(N-2) & \dots & y(N-p) & v(N-1) & v(N-2) & \dots & v(N-q) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \\ C_1 \\ C_2 \\ \vdots \\ C_q \end{bmatrix}$$

# Model Validation

$$\text{AIC} = -2\log L + 2d$$

$$\text{BIC} = -2\log L + d\log N$$

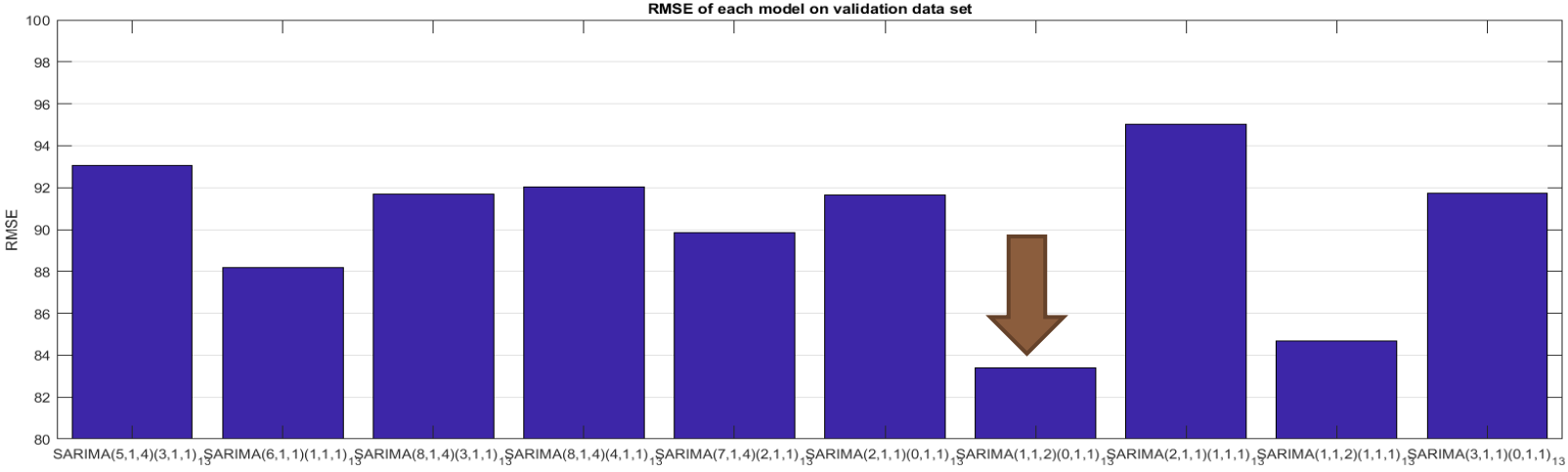
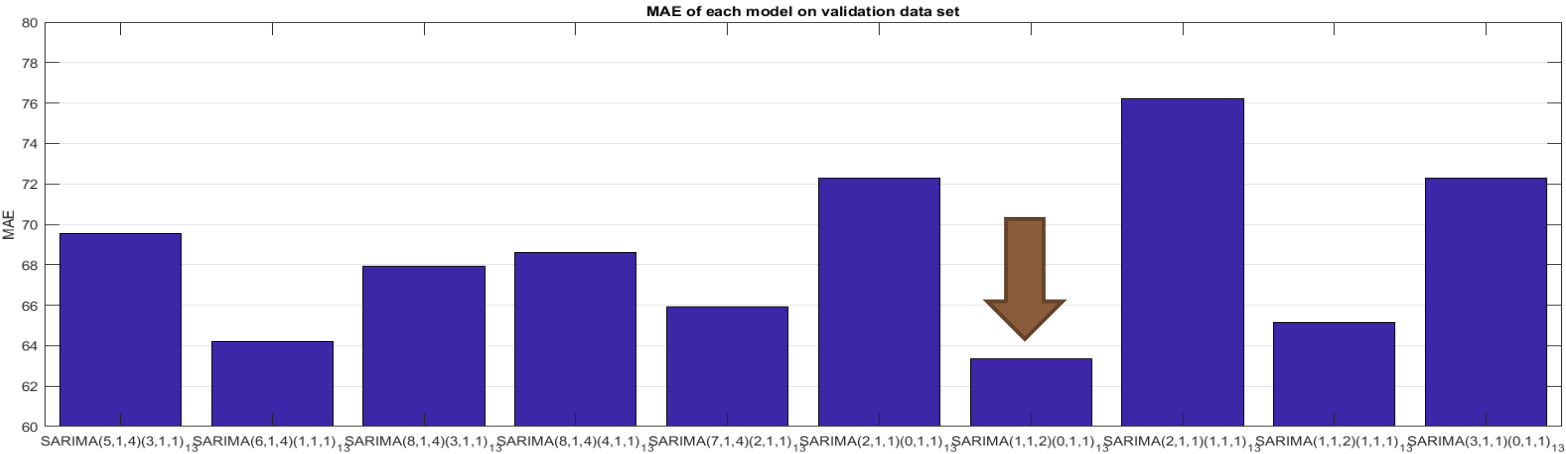
where,

$\log L$  is log-likelihood function

$d$  is number of parameters

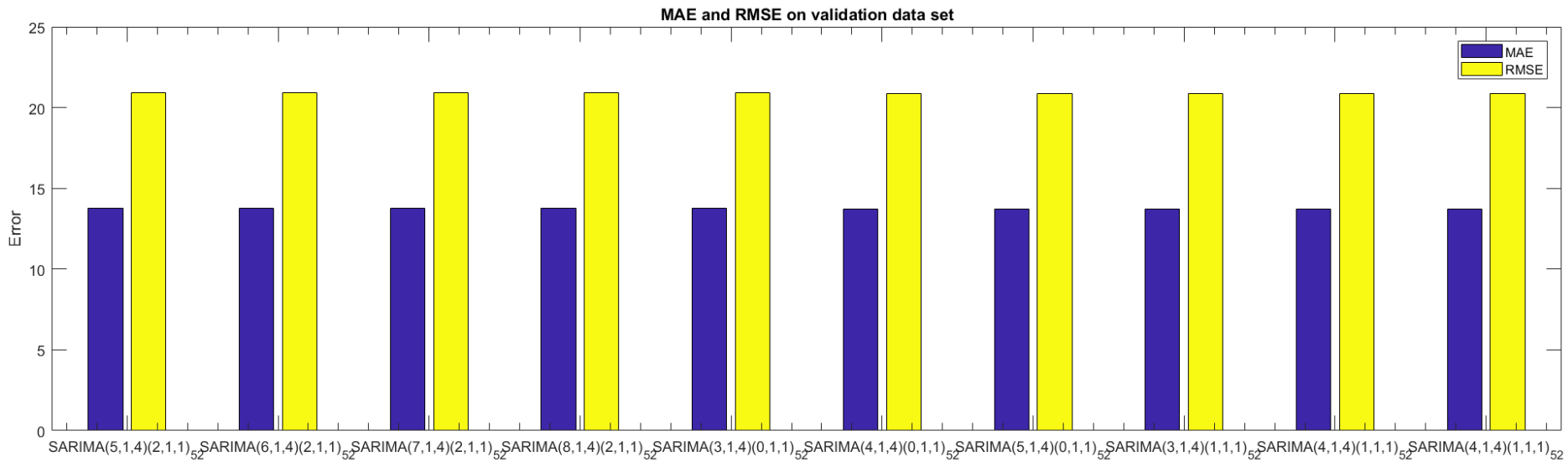
$N$  is number of data

# Error on validation data set of 60 minutes data set



Thus, this data use SARIMA(1,1,2)(0,1,1)<sub>13</sub>

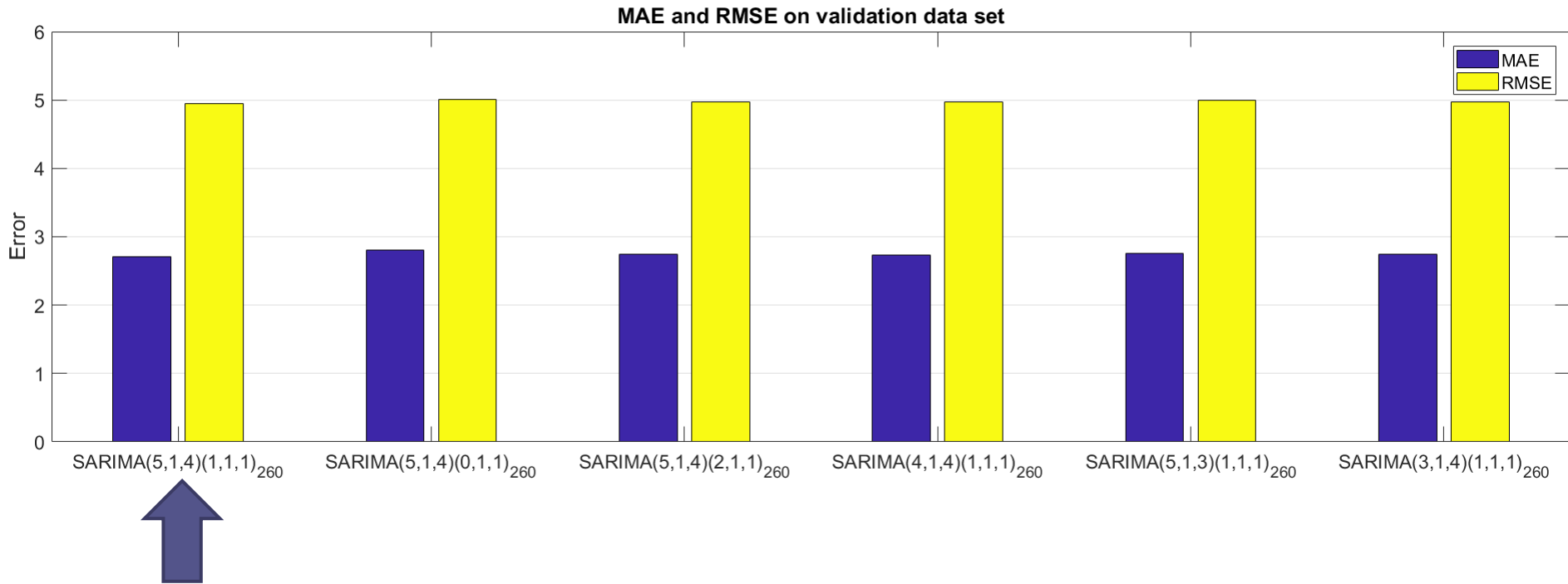
# Error on validation data set of 15 minutes data set



Thus, this data use SARIMA(4,1,4)(1,1,1)<sub>52</sub>



# Error on validation data set of 3 minutes data set



Thus, this data use SARIMA(5,1,4)(1,1,1)<sub>260</sub>

SARIMA(1,1,2)(0,1,1)<sub>13</sub> defined as

$$(1 - L^{13})(1 - 0.588L)(1 - L)I(t) = (1 - 0.878L - 0.113L^2)(1 - 0.933L^{13})e(t)$$

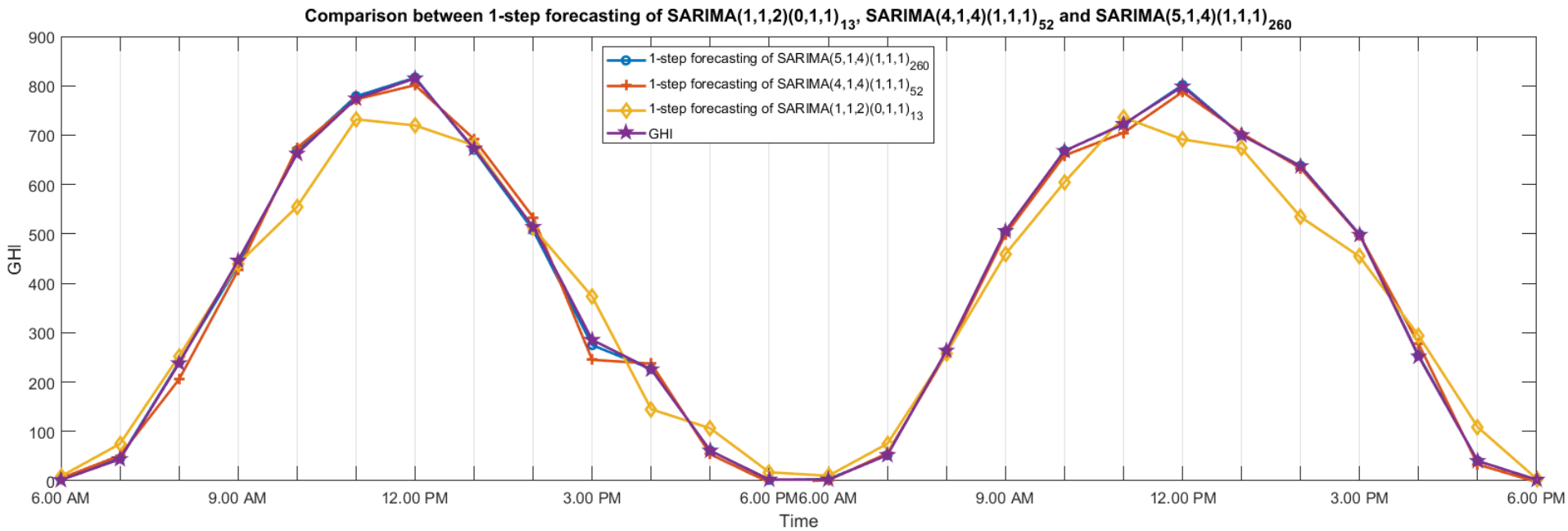
SARIMA(4,1,4)(1,1,1)<sub>52</sub> defined as

$$(1 - 0.011L^{52})(1 - L^{52})(1 - 0.880L + 0.179L^2 - 0.106L^3 - 0.033L^4)(1 - L)I(t) \\ = (1 - 0.338L + 0.111L^2 - 0.089L^3 - 0.673L^4)(1 - 0.947L^{52})e(t)$$

SARIMA(5,1,4)(1,1,1)<sub>260</sub> defined as

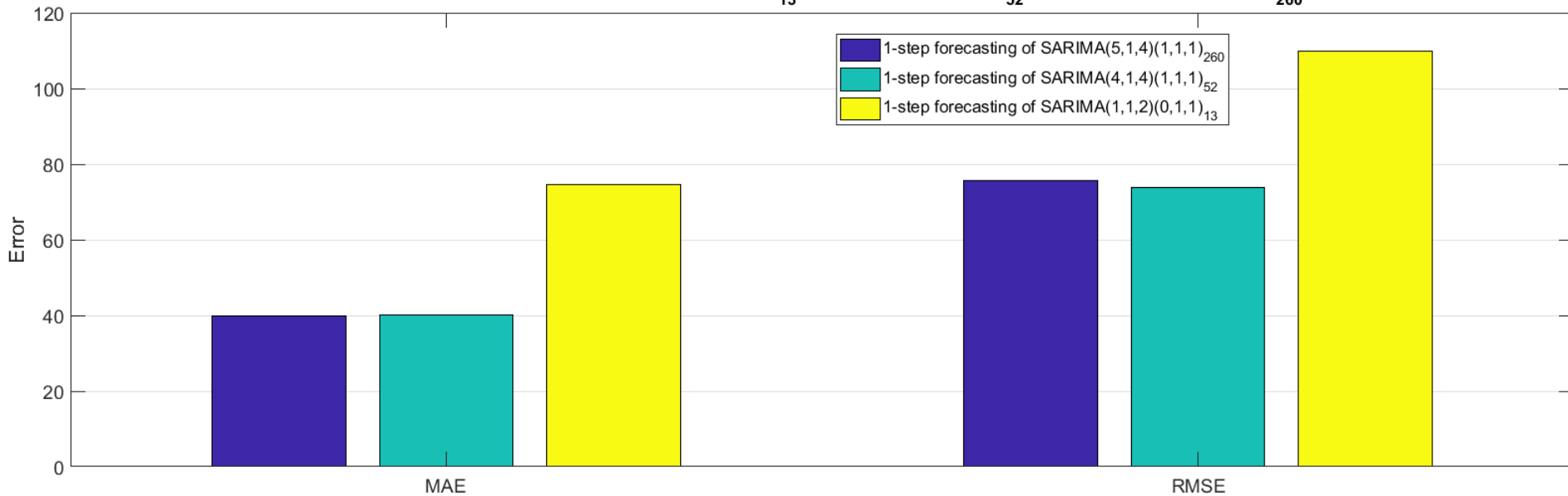
$$(1 - 0.034L^{260})(1 - L^{260})(1 - 2L + 0.829L^2 + 0.530L^3 - 0.298L^4 - 0.034L^4)(1 - L)I(t) \\ = (1 - 1.609L + 0.285L^2 + 0.565L^3 - 0.138L^4)(1 - 0.949L^{260})e(t)$$

# Forecasting Result

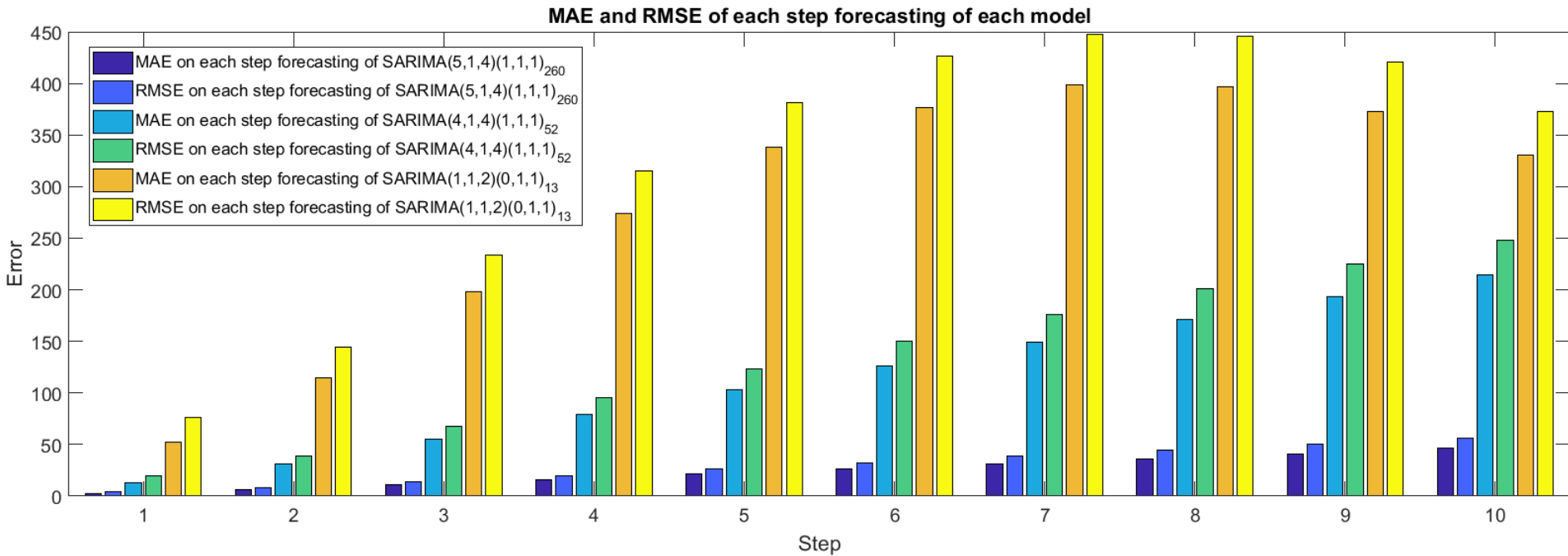


# Error on 1-step forecasting of each model

Comparison between 1-step forecasting of SARIMA(1,1,2)(0,1,1)<sub>13</sub>, SARIMA(4,1,4)(1,1,1)<sub>52</sub> and SARIMA(5,1,4)(1,1,1)<sub>260</sub> at same time index



# Forecasting Error



# Conclusion

# Q & A