SICE Annual Conference 2003

Fukui University

Feedback Stabilization of One-Link Flexible Robot Arms : An Infinite Dimensional System Approach.

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Outline

- Introduction
- Euler-Bernoulli beam equation
- Infinite-Dimensional System Theory
- The Closed-Loop System
- Stability Analysis
- Conclusion

Introduction

Recent Research

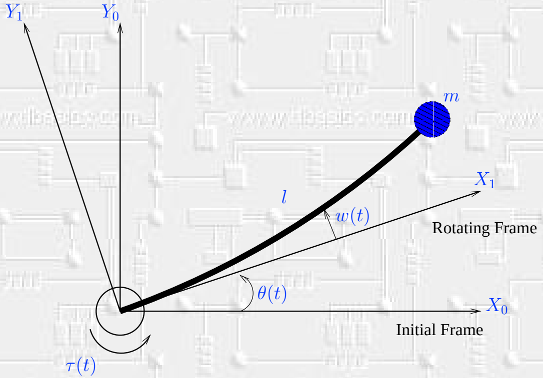
- Model : 1. Tip mass 2. Motor angle
- Control Law : velocity or its spatial higher derivative feedback.
- Stability Analysis : Spectral growth-determined condition, Energy Multiplier Method, Frequency domain condition.

The Objective of this work

✌ To understand the properties of the flexible robot arm system.

✌ To propose a control law that guarantees the closed-loop stability of the system.

Mathematic model of Flexible beam



$$\ddot{w}(x,t) + EIw''''(x,t) + x\ddot{\theta}(t) = 0 \tag{1}$$

$$\tau + EIw''(0,t) - I_{\mathrm{H}}\ddot{\theta} = 0 \tag{2}$$

$$m\left[\ddot{w}(l,t) + l\ddot{\theta}(t)\right] = EIw'''(l,t) \tag{3}$$

$$w(0) = w'(0) = w''(l) = 0 \tag{4}$$

Semigroup Theory

Consider an abstract Cauchy problem,

$$\dot{z}(t) = Az(t) + Bu(t), \quad t \geq 0 \tag{5}$$

$$z(0) = z_0 \in D(A) \tag{6}$$

where A is a closed operator with $D(A)$ dense in Z. The solution of (5)-(6) is,

$$z(t) = T(t)z_0 + \int_0^t T(t-s)u(s)ds \tag{7}$$

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Characterization of infinitesimal generator

Definition 1 $T(t)$ is a *contraction semigroup* if $\|T(t)\| < 1$, $\forall t \geq 0$

Theorem 2 Sufficient conditions for a closed, densely defined operator on a Hilbert space to be the infinitesimal generator of a C_0 semigroup satisfying $\|T(t)\| \leq e^{\omega t}$ are:

$$\mathrm{Re}\,\langle Az, z\rangle \;\leq\; \omega\|z\|^2 \quad \forall z \in D(A) \tag{8}$$

$$\mathrm{Re}\,\langle A^* z, z\rangle \;\leq\; \omega\|z\|^2 \quad \forall z \in D(A^*) \tag{9}$$

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Stability

1. $T(t)$ is *asymptotically stable* if

$$\|T(t)z\| \to 0 \quad \text{if} \quad t \to \infty \quad , \quad \forall z \in Z$$

2. $T(t)$ is *exponentially stable* if there exist $M \geq 1$ and $\omega > 0$ such that

$$\|T(t)\| \leq M e^{-\omega t}$$

3. $T(t)$ is *weakly stable* if $\forall x \, \forall y \in Z$

$$\langle T(t)x, y \rangle \to 0 \quad , \quad t \to \infty$$

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To prove the asymptotic stability

Theorem 3 Let $T(t)$ be a uniformly bounded semigroup on a Banach space X with the infinitesimal generator A and

1. $\sigma(A) \cap i\mathbb{R}$ is countable
2. $\sigma_P(A^*) = \emptyset$

then $T(t)$ is asymptotically stable.

Notation

- $H^m(0,l)$: Sobolev space order m with norm given by

$$\|u\|^2_{H^m} = \sum_{0\leq|\alpha|\leq m} \|D^\alpha u\|^2$$

- $H^2_0(0,l)$: $\{u \in H^2(0,l) \mid u(0) = u'(0) = 0\}$ with norm given by

$$\|u\|^2_{H^2_0} = \|u''\|^2$$

Result : $\|\cdot\|_{H^2_0} \sim \|\cdot\|_{H^2}$

The Closed-Loop System

We apply the control law

$$\tau(t) = -EIw''(0,t) + KI_{\mathrm{H}}\left[\rho\left\langle \dot{w}, x\right\rangle_H + ml\dot{w}(l,t)\right] \tag{10}$$

Substitute (10) in (2), the closed-loop equations are:

$$\ddot{w}(x,t) + \frac{EI}{\rho}w''''(x,t) = -xK\left[\rho\left\langle \dot{w}, x\right\rangle + ml\dot{w}(l,t)\right] \tag{11}$$

$$w(0,t) = w'(0,t) = w''(l,t) = 0 \tag{12}$$

$$m\ddot{w}(x,t) + mlK\left[\rho\left\langle \dot{w}, x\right\rangle + ml\dot{w}(l,t)\right] = EIw'''(l,t) \tag{13}$$

Problem formulation

Let $H = L_2(0,l)$ and consider the Hilbert space $\mathcal{H} = H_0^2(0,l) \oplus L_2(0,l) \oplus \mathbb{C}$ with an inner product

$$\langle u, v \rangle = EI \left\langle u_1'', v_1'' \right\rangle_H + \rho \left\langle u_2, v_2 \right\rangle_H + m \left\langle u_3, v_3 \right\rangle_{\mathbb{C}} \tag{14}$$

we can write (11)-(13) in the form $\dot{z} = \mathcal{A}z$, where

$$\mathcal{A} = \begin{bmatrix} 0 & I & 0 \\ -\frac{EI}{\rho}\frac{\partial^4}{\partial x^4} & -Kx\rho \left\langle \cdot, x \right\rangle & -Kxml \\ \frac{EI}{m}\frac{\partial^3}{\partial x^3}\,|_{x=l} & -Kl\rho \left\langle \cdot, x \right\rangle & -Klml \end{bmatrix} \tag{15}$$

$$D(\mathcal{A}) = \{(z_1, z_2, z_3) \in H^4(0,l) \oplus H_0^2(0,l) \oplus \mathbb{C} \mid z_1(0) = z_1'(0) = z_1''(l) = 0, z_2(l) = z_3\}$$

$$z(t) = \left[w(\cdot,t) \;\; \dot{w}(\cdot,t) \;\; \dot{w}(l,t) \right]^T \in \mathcal{H}$$

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$\mathcal{A}$ generates a C_0 semigroup

Lemma 4

1. $\mathcal{A}$ is the invertible and its inverse $\mathcal{A}^{-1} : \mathcal{H} \to \mathcal{H}$ is

$$\mathcal{A}^{-1}v = \begin{bmatrix} \frac{Kq_2(x)}{EI}[\rho \langle v_1, x\rangle + mlv_1(l)] - \frac{\rho}{EI}\int_0^x \int_0^{x_4} \int_{x_3}^l \int_{x_2}^l v_2(x_1)dx_1dx_2dx_3dx_4 + \frac{mq_1(x)}{EI}v_3 \\ v_1(x) \\ v_1(l) \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} q_1(x) &= \frac{x^3}{6} - \frac{lx^2}{2} \\ q_2(x) &= \rho\left(\frac{l^2x^3}{12} - \frac{l^3x^2}{6} - \frac{x^5}{120}\right) + mlq_1(x) \end{aligned}$$

2. $\mathcal{A}^{-1}$ is a bounded operator.
3. $\mathcal{A}$ is onto.
4. $\mathcal{A}$ is closed.
5. $0 \in \rho(\mathcal{A})$

Theorem 5 $\mathcal{A}$ generates a contraction semigroup.

proof. From the calculation,

$$\mathrm{Re}\left\langle \mathcal{A}u, u\right\rangle_{\mathcal{H}} = -K\left|\rho\left\langle u_2, x\right\rangle + mlu_3\right|^2 \leq 0 \tag{17}$$

$$\mathrm{Re}\left\langle \mathcal{A}^*u, u\right\rangle_{\mathcal{H}} = -K\left|\rho\left\langle u_2, x\right\rangle + mlu_3\right|^2 \leq 0 \tag{18}$$

The equations (8)-(9) are satisfied with $\omega = 0$ □

Stability Analysis

- The spectrum of the infinitesimal generator
- Eigenvalue analysis
- Closed-loop stability

The spectrum of the infinitesimal generator

To prove that the spectrum set consists of only the eigenvalues

Lemma 6 $\mathcal{A}^{-1}$ is compact.
Proof. By the Sobolev Imbedding and Arzela's theorem, see in paper.

Now we have,

- $\mathcal{A}$ is closed.
- $0 \in \rho(\mathcal{A})$
- $\mathcal{A}^{-1}$ is compact.

Then apply the following theorem,

Theorem 7 Let A be a closed linear operator with $0 \in \rho(A)$ and A^{-1} compact. The spectrum of A consists of only isolated eigenvalues with finite multiplicity.

The eigenvalues

We will show that all eigenvalues lie in the open LHP. Consider the eigenvalue problem

$$\mathcal{A}\phi(x) = \lambda\phi(x) \tag{19}$$

where λ and $\phi(x) = \begin{bmatrix}\phi_1(x) & \phi_2(x) & \phi_3\end{bmatrix}^T$ be an eigenvalue and the corresponding eigenvector of $\mathcal{A}$.

$$\phi_1''''(x) + \frac{\rho\lambda^2}{EI}\phi_1(x) = -\frac{\rho K}{EI}\lambda\left[\rho\left\langle\phi_1, x\right\rangle + ml\phi_1(l)\right]\cdot x \tag{20}$$

$$\phi_1(0) = \phi_1'(0) = \phi_1''(l) = 0 \tag{21}$$

$$\phi_1'''(l) = \frac{Kml}{EI}\lambda\left[\rho\left\langle\phi_1, x\right\rangle + ml\phi_1(l)\right] + \frac{m}{EI}\lambda^2\phi_1(l) \tag{22}$$

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Take the inner product with ϕ_1 on both sides in (20)

$$\langle \phi_1'''', \phi_1 \rangle + \frac{\rho\lambda^2}{EI} \langle \phi_1, \phi_1 \rangle + \frac{\rho K \lambda}{EI} (\rho \langle \phi_1, x \rangle + ml\phi_1(l)) \langle x, \phi_1 \rangle = 0 \tag{23}$$

since

$$\langle \phi_1'''', \phi_1 \rangle = \lambda \frac{\rho Kml}{EI} \langle \phi_1, x \rangle \overline{\phi_1(l)} + \lambda \frac{Km^2 l^2}{EI} |\phi_1(l)|^2 + \lambda^2 \frac{m}{EI} |\phi_1(l)|^2 + \|\phi''\|^2 \tag{24}$$

substitute in (23), we get

$$\lambda^2 \left\{ m|\phi_1(l)|^2 + \rho \|\phi_1\|^2 \right\} + \lambda K \left| \rho \langle \phi_1, x \rangle + ml\phi_1(l) \right|^2 + EI\|\phi''\|^2 = 0 \tag{25}$$

Let $\lambda = a + ib$, (25) can be split into two equations.

$$(a^2 - b^2)(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) + a \cdot K \left| \rho \langle \phi_1, x \rangle + ml\phi_1(l) \right|^2 + EI\|\phi''\|^2 = 0 \tag{26}$$

$$2ab(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) + b \cdot K \left| \rho \langle \phi_1, x \rangle + ml\phi_1(l) \right|^2 = 0 \tag{27}$$

It can be shown that

$$|\rho \langle \phi_1, x \rangle + ml\phi_1(l)|$$

is not equal to zero.
Therefore, from (26), if $b = 0$ then

$$a^2(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) + a \cdot K \left|\rho \langle \phi_1, x \rangle + ml\phi_1(l)\right|^2 + EI\|\phi''\|^2 = 0$$

All coefficients of the polynomial a are all positive. Thus $a < 0$.
From (27), if $b \neq 0$ then

$$a = -\frac{K \left|\rho \langle \phi_1, x \rangle + ml\phi_1(l)\right|^2}{2(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2)} < 0$$

Therefore $\mathrm{Re}(\lambda) < 0$.

Closed-Loop Stability

- ✔ $\sigma(\mathcal{A}) = \sigma_P(\mathcal{A})$
- ✔ The real part of all eigenvalues are negative.
- ✔ $\sigma(\mathcal{A}) \cup i\mathbb{R} \Longrightarrow$ is countable.
- ✔ $\sigma_P(\mathcal{A}^*) = \sigma_r(\mathcal{A}) = \emptyset$
- ✔ A contraction semigroup is uniformly bounded.
- ✔ From theorem 3, the semigroup is asymptotically stable.

Conclusions

- Feedback control signal through motor acceleration.
- The Proposed control law is the sum of the tip deflection and its linear functional.
- The infinitesimal generator of the closed-loop system generates a contractions semigroup.
- The spectrum consists of only the eigenvalues.
- All eigenvalues have negative real parts.
- The closed-loop system is asymptotically stable.